# Quantum Cryptography Without Basis Switching

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Listen, do you want to know a secret? Do you promise not to tell?

- *John Lennon and Paul McCartney*

# Declaration

This thesis is an account of research undertaken with the supervision of Dr Ping Koy Lam, Dr Tim Ralph and Dr Warwick Bowen between February 2004 and October 2004. It is a partial fulfilment of the requirements for the degree of a Bachelor of Science with Honours in theoretical physics at the Australian National University, Canberra, Australia.

Except where acknowledged in the customary manner, the material presented in this thesis is, to the best of my knowledge, original and has not been submitted in whole or part for a degree in any university.

Christian Weedbrook
29th October 2004

# Acknowledgements

> During the three years of residency, it was the relationships that got you through.
>
> - *J.D., Scrubs*

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*This thesis is dedicated to Alexandra.*

# Abstract

Quantum cryptography is one of the most interesting and popular fields in quantum information today. Its chief benefit lies in exploiting principles of quantum mechanics to offer absolute security in the transmission of information. This thesis introduces a new continuous variable quantum cryptographic protocol. The main difference between our protocol and all previous ones, is the elimination of switching between measurement bases - a step that was thought crucial in maintaining security. Our theoretical analysis shows that by eliminating switching of measurement bases from the protocol, we still maintain the required security, but also achieve significantly higher secret key rates than any other known continuous variable quantum cryptographic protocol.

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Chapter 1

# Introduction

O-Ren Ishii: You didn't think it was gonna be that easy, did you?
The Bride: You know, for a second there, yeah, I kinda did.

- *KILL BILL Vol.1*

## 1.1 Overview

The desire to communicate in private has always been apart of human civilization. Although its origins are unknown, it would almost certainly have started not long after the beginning of literacy. The art of communicating in secret, between a sender and receiver, is known as cryptography [1]. It has a long and fascinating history, full of exciting and interesting stories: the assassination of a Queen, the world of espionage, the secret hiding and disguising of messages, Enigma machines and much more [2, 1]. Cryptography usually consists of encrypting a message with a secret key. As a result the message is then scrambled and is only able to be read by the receiver, who also has a key. For this reason, the key is kept private. However, cryptography was changed in the 1970's, when it was discovered that you could publicly distribute your key and yet still maintain secure communications. An example of this public key distribution is the RSA code, which forms the basis of all of today's crypto-communication systems. Since the very beginning, there has been a back and forth struggle between the code-maker and code-breaker to remain dominate. However, in 1984 a new advancement in cryptography might have ended the struggle for good, in favor of the code-maker. A new theory called quantum mechanics is the reason why.

Since its beginnings in the early twentieth century, quantum mechanics [3] has developed a successful theoretical framework for the world around us. It is a theory based on the atomic or microscopic world, and is probabilistic in nature. Quantum optics [4] is a branch of quantum theory, that deals with the description and controlling of optical light fields. It illustrates the differences between our everyday "classical" world and the weird and wonderful quantum world, by revealing such non-classical features as squeezing [5, 6] and entanglement [7]. During the last few decades, quantum physics, along with quantum optics, have come together to form new ways of processing information. This is known as quantum information [8].

Quantum information is currently receiving enormous interest and success in both theoretical and experimental physics. It is a combination of quantum physics with computer science and mathematics, with a goal of providing more efficient and secure ways of transmitting and processing information. Quantum information theory has lead to such exciting and diverse topics as teleportation [9], quantum computation [10, 11], quantum error correction [12], dense coding [13] and quantum search algorithms [14]. These applications of quantum mechanics were originally developed using discrete variables. These discrete systems deal with the manipulation of quantum bits (qubits) or single photon states. However there has been a move towards continuous variables, which are easier to generate and offer higher bandwidths and higher quantum efficiency production in the measuring of quantum states. This has lead to continuous variable quantum information theory [15]. The main incentive for this transfer, is due to more practical considerations in quantum information protocols. The detecting of discrete single photons has an efficiency of about 60%, whereas the continuous systems have an efficiency of near unity. This thesis will be concerned with continuous variable systems and in particular, continuous variable quantum cryptography.

Quantum cryptography [16] is the science of sending secret messages using properties of quantum mechanics. This property is namely the inability to simultaneously measure two non-commuting observables perfectly. As it is these observables that the information is encoded onto, any eavesdropper will ultimately disturb the quantum system and therefore add additional noise. Consequently a sender and receiver are able to generate a correlated secure key; provided this disturbance or noise is below a certain allowable level. Quantum cryptography could have been invented as soon as quantum mechanics was discovered and understood in the first few decades of the 20th century. However it was only revealed in 1984 [17] after an unpublished paper in 1970 [18]. Now quantum cryptography offers the possibility of 100% security against any eavesdropper. Finally ending the battle between the code-maker and the code-breaker. The first version of quantum cryptography, or quantum key distribution (as it is also known), was using discrete photon polarizations. In 1999 a continuous variable version was formed which offered a number of benefits over the discrete regime [19]. All forms of quantum cryptography had what seemed an inherent step in their protocols. The switching of measurement bases. For over 20 years now, switching has appeared in all quantum key distribution protocols and it was thought that the security of these protocols was due to this switching. This leads to the two main aims of this thesis:

- *Do quantum cryptographic protocols require switching?*
  Switching refers to a technique used by the receiver, to randomly switch the measurement bases. It has long been a part of both the discrete and continuous variable schemes of quantum cryptography. It was thought that switching offers the necessary condition for security in these protocols. We will investigate whether switching is absolutely necessary and if we can create a new continuous variable quantum cryptography protocol that resists the need to randomly switch between measurement bases.

- *If no, then how does it compare to previous protocols?*
  The process of switching in the continuous variable regimes limits the bandwidth in the cryptographic protocols. Therefore the elimination of switching would offer practical advantages: simplicity and higher bandwidths. In fact, our results [20] in Chapter 5 will show that switching, which has existed for over two decades now, is no longer necessary. The protocol that we have developed, known as the SQM protocol, is also applicable to previous coherent state continuous variable quantum cryptographic protocols.

## 1.2 Thesis Structure

This thesis consists of five main chapters. Chapter 2 introduces the major theoretical concepts that will be used in this thesis. These concepts deal with the foundations of quantum mechanics and the fundamentals of quantum optics. Chapter 3 introduces classical cryptography: its history, the idea of key distributions (both private and public) and how an application of quantum theory, in the form of quantum computers, provides insecurities in today's classical encryption protocols. Chapter 4 introduces quantum cryptography and begins with its origins in quantum money. We then discuss the two main forms of quantum cryptography: discrete and continuous. Key distillation and privacy amplification are then introduced with a final comment on the experimental versions of quantum cryptography. Chapter 5 consists of the original research that was undertaken in this thesis and introduces our new quantum cryptographic protocol. Chapter 6 ends with concluding remarks along with the future prospects of both quantum cryptography and quantum information in general.

## 1.3 Publication

A publication has resulted from the work that was carried out in this thesis. The original research, which appears in Chapter 5, has been published in the following journal article:

- *Quantum Cryptography Without Switching*,
  Christian Weedbrook, Andrew Lance, Warwick Bowen, Thomas Symul, Tim Ralph and Ping Koy Lam,
  Physical Review Letters **93**, 170504 (2004).

Chapter 2

# Quantum Theory

> A newspaper can be read and you don't hurt it by reading it. For really small things, the process of observing them disturbs them.
>
> - *Charles Bennett*

> Dr. Emmett Brown: 1.21 gigawatts? 1.21 gigawatts? Great Scott!
> Marty McFly: What the hell is a gigawatt?
>
> - *Back to the Future*

This chapter introduces a theoretical framework that will be used throughout this thesis. We introduce such fundamental concepts as the uncertainty principle, phase space representation and detecting quantum states of light.

## 2.1 The Uncertainty Principle

> We cannot know, as a matter of principle, the present in all its details.
>
> - *Werner Heisenberg*

Quantum mechanics is one of the most successful theories in the history of science. It is a probabilistic theorem, which as a result, often deals with uncertainties and statistics. One concept, the uncertainty relation, illustrates this view of quantum mechanics and can be derived using commutators. In quantum theory, the commutator of two arbitrary observables[1], $\hat{A}$ and $\hat{B}$, is a mathematical operation given by

$$[\hat{A}, \hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A} \tag{2.1}$$

The operators $\hat{A}$ and $\hat{B}$ are said to commute when $[\hat{A}, \hat{B}] = 0$ and are non-commuting when $[\hat{A}, \hat{B}] \neq 0$. One physical interpretation of two observables commuting is that we can determine their values exactly, i.e. without any uncertainty. However if they are non-commuting then it is impossible to determine them both simultaneously, without

[1] An observable corresponds to the eigenvalue of its operator. It is hermitian since it has real expectation values.

any uncertainty or error. One such example is say, $[\hat{A}, \hat{B}] = 2iC$, where C is a complex constant. Then the product of these uncertainties is given by

$$\Delta\hat{A}\Delta\hat{B} \geq \langle |C| \rangle \tag{2.2}$$

where $\Delta\hat{A}$ and $\Delta\hat{B}$ are the uncertainties. Equation (2.2) is known as the Heisenberg uncertainty principle . By taking the square of these uncertainties or standard deviations given in Eq. (2.2), we end up with

$$V_A V_B \geq C^2 \tag{2.3}$$

where $V_A = \langle(\Delta\hat{A})^2\rangle$ and $V_B = \langle(\Delta\hat{B})^2\rangle$. The standard deviation for any arbitrary operator $\hat{D}$ is defined as

$$\Delta\hat{D} = \sqrt{\langle\hat{D}^2\rangle - \langle\hat{D}\rangle^2} \tag{2.4}$$

The uncertainty relation is an important consequence of quantum mechanics and can be used, along with the non-Hermitian boson annihilation and creation operators, to give an understanding of a quantum system. The annihilation operator $\hat{a}$ and creation operator $\hat{a}^\dagger$ are mathematical functions that can describe the light field as either a function of time $\hat{a}(t)$ or frequency $\hat{a}(\omega)$. The operators for this thesis are assumed to be a function of time. The boson commutator relation of these two field operators is

$$[\hat{a}, \hat{a}^\dagger] = 1 \tag{2.5}$$

where $\hat{a} = \frac{1}{2}(\hat{X}^+ + i\hat{X}^-)$ [5]. We can define a new set of operators, known as quadratures, from the annihilation and creation operators

$$\begin{aligned} \hat{X}^+ &= \hat{a}^\dagger + \hat{a} && (2.6) \\ \hat{X}^- &= i(\hat{a}^\dagger - \hat{a}) && (2.7) \end{aligned}$$

Here $\hat{X}^+$ and $\hat{X}^-$ correspond to the amplitude and phase of an electric field, respectively, and are analogous to position and momentum in classical physics. The commutator of these quadratures is given by

$$[\hat{X}^+, \hat{X}^-] = 2i \tag{2.8}$$

Using Eq. (2.2), this leads to uncertainty and variance relations of

$$\begin{aligned} \Delta\hat{X}^+\Delta\hat{X}^- &\geq 1 && (2.9) \\ V^+V^- &\geq 1 && (2.10) \end{aligned}$$

This shows us that we cannot simultaneously measure the amplitude and phase quadratures precisely or without assuming some sort of penalty. Equation (2.10) has a lower limit of 1. This lower bound is known as the quantum noise limit or the shot noise limit.

## 2.2 Phase Space Representation

The phase space gives a two dimensional representation of the two orthogonal quadratures, $\hat{X}^+$ and $\hat{X}^-$, that describe a continuous variable light field. This diagrammatic scheme

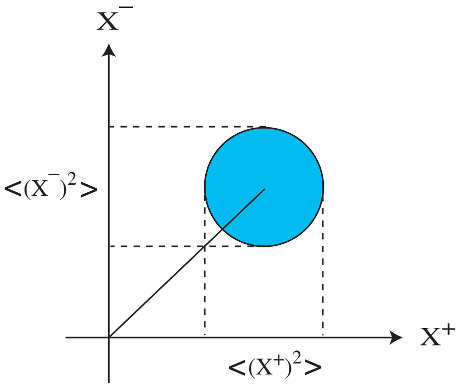


**Figure 2.1**: Phase space or “ball and stick” representation of the light field.

can also be viewed as a “ball and stick” picture (see Fig. 2.1). The length of the stick is the classical field amplitude and the ball represents the quantum fluctuations or uncertainty in both quadratures. So when the quantum uncertainty is much less than the classical amplitude and the average fluctuations are zero, we can linearize the field operators using

$$\hat{a} = \alpha + \delta\hat{a} \tag{2.11}$$

$$\hat{a}^\dagger = \alpha^* + \delta\hat{a}^\dagger \tag{2.12}$$

where $\alpha$ is a c-number representing the steady state or amplitude and $\delta\hat{a}$ are the quantum fluctuations (so we are simply defining $\delta\hat{a} = \hat{a} - \langle\hat{a}\rangle$). Thus in the phase space representation the stick is the steady state or classical amplitude and the ball is the quantum fluctuations. The diameter of the ball is regarded as the variance of each quadrature. Two common states of light that can be represented on phase space are the coherent state and the squeezed state.

### 2.2.1 Coherent States

A coherent state is the most quasi-classical of all quantum states. This is because it is a minimum uncertainty state that lower bounds the inequality given in Eq. (2.9), i.e. $V^+V^- = 1$. Figure (2.1) shows a coherent state as represented by a phase space diagram. A coherent state $|\alpha\rangle$ can be written as a sum of number or Fock states $|n\rangle$

$$|\alpha\rangle = e^{-\frac{|\alpha^2|}{2}} \sum_n^\infty \frac{\alpha^n}{\sqrt{n!}} |n\rangle \tag{2.13}$$

where all possible Fock states can be generated by repeatedly applying the annihilation operator on a vacuum state $|0\rangle$. This is defined mathematically as

$$|n\rangle = \frac{(\hat{a}^\dagger)^n}{\sqrt{n!}} |0\rangle \tag{2.14}$$

A coherent state has the following properties

$$\hat{a}|\alpha\rangle = \alpha|\alpha\rangle \tag{2.15}$$

$$|\alpha\rangle = \hat{D}|0\rangle \tag{2.16}$$

where $\hat{D} = \exp(\alpha\hat{\mathrm{a}}^\dagger - \alpha^*\hat{\mathrm{a}})$ is called the displacement operator. Therefore the coherent state is the eigenstate of the annihilation operator and is obtained by acting the displacement operator on the vacuum state. A type of coherent state that is worth mentioning is the vacuum state $|0\rangle$. It has the same quantum fluctuations as a coherent state but with an average number of photons equal to zero. So on the "ball and stick" picture it would be a ball centered at the origin with no stick (see Fig. 2.2). One place that the vacuum state enters into our analysis is through a beam splitter (see below).

### 2.2.2 Squeezed State

Squeezing is when one of the uncertainties in Eq. (2.9) is reduced ("squeezed") below the quantum noise limit (Fig. 2.2 shows a squeezed state in phase space). In order for this inequality to still hold, the other uncertainty is increased. Amplitude squeezing is when $V^+ < 1$ and $V^- > 1$ and phase squeezing is when $V^- < 1$ and $V^+ > 1$. Perfect squeezing in the amplitude quadrature occurs when $V^+ \rightarrow 0$ resulting in the phase quadrature $V^- \rightarrow \infty$. Anti-squeezing results when one of the quadratures is greater than 1. The amount of squeezing is characterized by the squeezing parameter $\mathrm{s}^\pm$, where $0 \leq s^\pm \leq 1$. Understanding that the uncertainty relation between the squeezing parameter and the variable must hold, leads to

$$\mathrm{s}^\pm \geq \frac{1}{\mathrm{V}^\mp} \tag{2.17}$$

which comes from the fact that $V^+V^- = 1$. The creation of squeezed light [21, 22] can be useful in the generation of entanglement and such applications as gravitational wave detection and quantum cryptography.

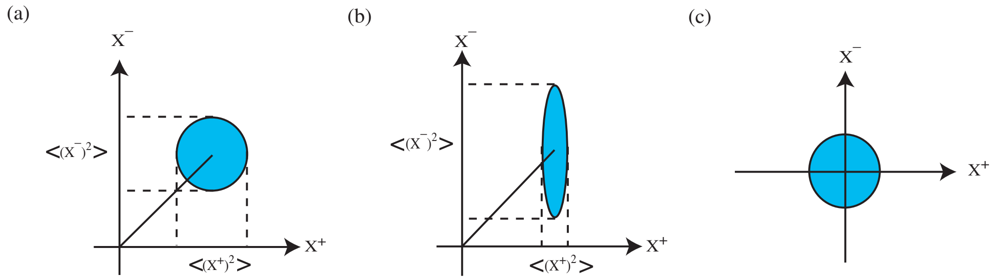


**Figure 2.2:** Three types of phase space representations. (a) Coherent State (b) A squeezed state (amplitude squeezed) (c) Vacuum state.

## 2.3 Theoretical Concepts for Experiments

A number of the theoretical concepts in this thesis are based on experimental techniques, including the use of beam splitters (and the vacuum noise associated with it) and the measuring processes that are needed to understand and analyze the incoming light field.

### 2.3.1 Beam Splitters and Vacuum Noise

> Dr. Egon Spengler : Wait! Wait! There's something I forgot to tell you.
> Dr. Peter Venkman : What?
> Dr. Egon Spengler : Don't cross the streams.
> Dr. Peter Venkman : Why not?
> Dr. Egon Spengler : Trust me. It would be bad.
>
> \- *Ghostbusters*

A beam splitter is one of the most commonly used tools in applications relating to quantum information. It is an optical device that splits an incoming light beam into two beams: one reflected beam and one transmitted beam. One feature of a beam splitter is the coupling in of vacuum noise with the incoming light. This influx of vacuum noise is a quantum mechanical feature that is necessary in order to preserve, among other things, the uncertainty principle and the bosonic commutator relations. In Fig. 2.3, an incoming light beam, characterized by the boson field operator $\hat{a}$, hits a beam splitter in which vacuum noise $\hat{b}$ has entered. The two output operators are given by the following beam splitter equations

$$\hat{d} = \sqrt{\eta}\,\hat{a} + \sqrt{1-\eta}\,\hat{b} \qquad (2.18)$$

$$\hat{c} = \sqrt{1-\eta}\,\hat{a} - \sqrt{\eta}\,\hat{b} \qquad (2.19)$$

where $\eta$ is the transmission coefficient that ranges in value from 0 to 1: with 0 indicating zero transmission and 100% refection and with 1 signifying the complement. It is important to note that when we take the variance of Eq. (2.18), the variance of the vacuum noise will be normalized to unity, i.e. $V_b = \langle(\hat{b})^2\rangle = 1$.

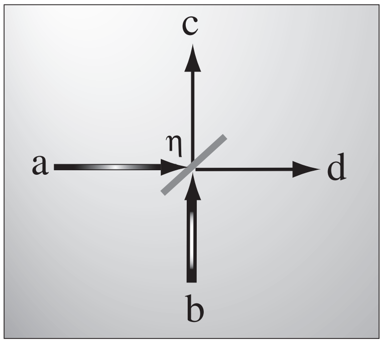


**Figure 2.3**: Schematic diagram of a beam splitter

### 2.3.2 Detecting Quantum States of Light

The theory of measurement processes is an important part of any analysis involving quantum theory. We have already introduced variances and quadratures, which are all important theoretical concepts. But how do we go about measuring them? This process consists of a detector that transforms the incoming beam of light into a photocurrent which is transformed into statistical data via a spectrum analyzer. Two such processes include direct detection and standard homodyne detection.

### Direct Detection

Direct detection (see Fig. 2.4) is a simple optical measurement whose photocurrent I(t) is proportional to the intensity of the light field

$$\mathrm{I(t)} \propto |\alpha|^2 + \alpha\, \delta \mathrm{X}^{+}(\mathrm{t}) \tag{2.20}$$

where $\alpha$ is the classical amplitude. This shows us that by measuring this direct detection photocurrent we can, in principle, measure the amplitude quadrature $\Delta \mathrm{X}^{+}(\mathrm{t})$ to an arbitrary precise value. The phase quadrature measurement can not be obtained by this method. This is due to the destruction of the phase as a result of the rapid oscillation of the electric field with respect to the electric current. The phase quadrature can be measured using standard homodyne detection.

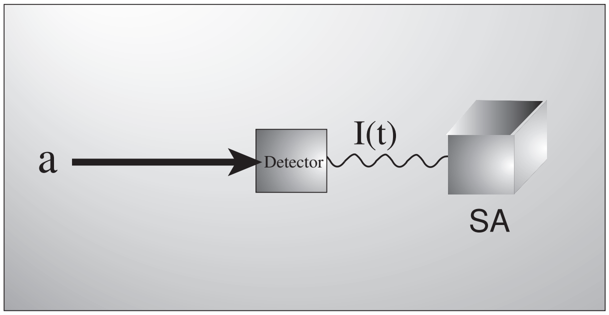


**Figure 2.4:** Schematic diagram of direct detection. An incoming light field, described by the operator $\hat{a}$, hits a detector. The light is then transformed into a photocurrent I(t), which can be studied by a spectrum analyzer.

### Standard Homodyne Detection

A homodyne detector is an experimental tool used to measure either the amplitude or phase quadratures of a light field (see Fig. 2.5). Self-homodyning is when the incoming light is interacted with the vacuum noise that enters into the other unused port of the beam splitter. We can then take the sum (difference) of the photocurrents to measure the amplitude of the incoming signal beam (vacuum noise). Homodyning with a local oscillator[2] (LO) is a form of optical mixing that allows us to determine the phase quadrature. The local oscillator is a bright light (large amplitude) of a much higher photon number than the incoming signal (sig). By taking the sum (s) and difference (d) of the photocurrents we arrive at

$$I(t)^s \propto \beta\delta X^{+}_{LO} \tag{2.21}$$
$$I(t)^d \propto \beta\delta X^{(\theta+\pi/2)} \tag{2.22}$$

where $\beta$ is the steady state of the local oscillator, $\theta$ is the phase between the local oscillator and signal beam, and we have only included the quantum fluctuations. From this we see

[2] The phase of something is only known with respect to something else. Thus a local oscillator is a bright beam of light with a known phase that acts like a clock or reference point in which other beams can then be compared to.

that by taking the sum of the photocurrent we can obtain the amplitude of the local oscillator and the difference gives us the phase of the signal beam. However in later chapters of this thesis we will refer to another type of detection known as simultaneous quadrature measurements. This is essentially double homodyne detection where the switching between the two quadratures is not required.

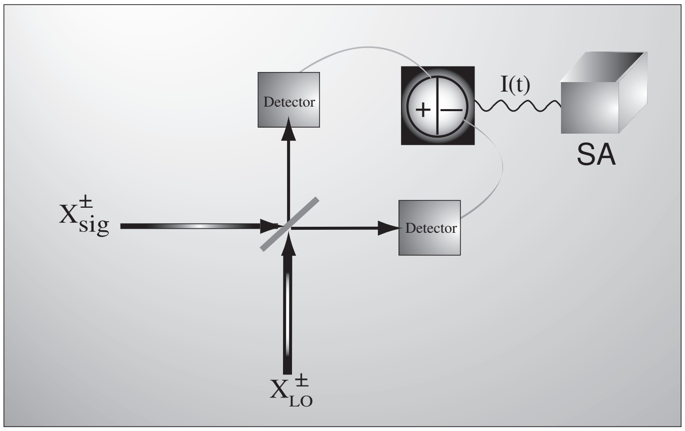


**Figure 2.5:** Schematic diagram of homodyne detection. Two light beams, a signal $X^{\pm}_{sig}$ and local oscillator $X^{\pm}_{LO}$, are combined on a beam splitter. The following output beams are them measured using two detectors. The sum (+) or difference (-) of these detectors can then be taken and the resulting photocurrent I(t) is studied via a spectrum analyzer.

## 2.4 Conclusion

This Chapter has introduced the theoretical concepts needed to understand the subsequent analysis used in this thesis. These include such quantum mechanical formalisms as the uncertainty principle and commutators, along with the theory associated with the description of light: quadratures, phase space representation. Beam splitters, vacuum noise and standard measuring techniques have also been defined as a way of using theoretical concepts mirroring experimental techniques.

# Classical Cryptography

> Joey: Hey Chandler, when you see Frankie tell him that Joey says hi. He'll know what it means.
> Chandler: Gee, I don't know. Do you think he'll be able to crack your code?
>
> \- *Friends*

This chapter introduces the concept of classical cryptography and its now famous protagonists, Alice and Bob, along with its leading antagonist, Eve. It also goes over a brief history of cryptography and its various forms. The concept of key distribution - private and public - is then introduced. Finally we discuss Shor's algorithm which shows an insecurity in the current encryption programs that could be exploited by a quantum computer. This leads into what the remainder of the thesis is essentially about: a specific application of quantum physics that leads to secure transmissions.

## 3.1 Introduction

Classical cryptography [1] is the encrypting of information by a sender so it can only be understood by the intended receiver. It has existed for thousands of years and is part of the broader field of cryptology, which also includes cryptanalysis - the breaking or decrypting of codes. The main characters in cryptography are Alice, Bob and Eve. The sending of information is usually from point A (Alice the sender) to point B (Bob the receiver). Eve is the typical villain who eavesdrops into Alice and Bob's communications. The process of cryptography usually consists of Alice encrypting a readable message, or plaintext, by using a secret algorithm, known as the key. The plaintext can then be scrambled and only read by Bob, who is also in possession of the key and is therefore able to decrypt the message (see Fig. 3.1). The success of any cryptographic protocol relies on the security of the key. If Eve was able to determine the key then she would also be able to read Alice and Bob's messages. This thesis will be primarily concerned with the science of cryptography but, as we will see shortly, it could also be known as the science of key distribution. This is due to the fact that encrypting information is easy - how to securely deliver or build a key between Alice and Bob - that's the hard part. The exact origin of cryptography or key distribution is unknown. However, what is known, is the importance of cryptographic protocols in maintaining security in this age of information.

[1] The words classical cryptography and cryptography will be used interchangeably.

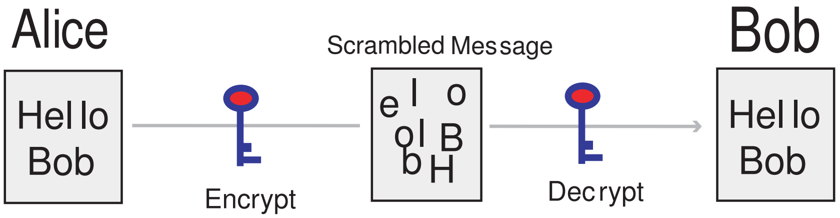


**Figure 3.1**: The process of cryptography

## 3.2 A Brief History of Cryptography

> Detective stories or crossword puzzles cater for the majority; the solution of secret codes may be the pursuit of a few.
>
> - *John Chadwick*

Governments and defence forces world wide have long been interested in cryptology as a way of keeping their secrets and breaking everyone elses secrets. So it is not surprising that one of the first recorded uses of cryptography was by the military. The scytale was a simple but effective cryptographic device that was known to have been used by the Greek army around 400 BC. The scytale was a solid wooden rod of specific dimensions that had a piece of long material wrapped around it. The message was then written across the material and then unravelled to reveal a long strip of material with a scrambled message. It was then delivered to the intended recipient who owned the same dimensioned scytale and was therefore able to unscramble the message. This form of cryptography is called transposition, as the message is transposed or jumbled around from its original state.

Julius Caesar also used cryptography for military purposes and his particular technique involved a simple letter substitution. This became known as the Caesar cipher or simply, substitution. For example, suppose we want to encrypt the message **THE CODE BOOK**. Alice and Bob would agree on a specific substitution before hand, which Alice would use to encode her message and Bob would use to decrypt the message. Below is one such substitution.

| | |
|---|---|
| normal alphabet | a b c d e f g h i j k l m n o p q r s t u v w x y z |
| cypher alphabet | r s t u v w x y z a b c d e f g h i j k l m n o p q |
| | |
| normal word | THE CODE BOOK |
| cypher word | KYV TFUV SFFB |

So whenever you see an **a** it becomes an **r**, **b** becomes an **s** and so forth. Therefore the message **THE CODE BOOK** is encrypted using the cypher alphabet and it is now read as **KYV TFUV SFFB**. So unless you have access to the cypher alphabet then you are unable to read the message [2]. Julius Caesar's encryption involved the substitution of

[2] This is not entirely true. There are a number of ways to break most encryptions. Frequency analysis is often used to break such codes, however this technique and others, are beyond the scope of this thesis. For a more detailed discussion of frequency analysis and a great starting point for cryptography see [1].

Roman letters with Greek letters, which was enough to ensure his message did not fall into his enemy's hands.

Another form of cryptography is steganography. This form of security involved the physical hiding of a message from the enemy. Ancient cultures would shave a messenger's head and write a note on it. They would then wait for the hair to grow back and send him to the receiver who would again shave his head to reveal the message. Clearly not the most time saving of all cryptographic protocols! Another example of steganography occurred during World War II. A secret document would be shrunk to the size of a full stop and then hidden over the top of a full stop on a typed page. A lot of encryption techniques would inevitably use a combination of say, substitution and transposition, in order to maximize the message's security. The scytale could also used as a steganographic device, sometimes being disguised as a belt around the messenger. The document that was shrunk to the size of a full stop could also have been encrypted before it was reduced, by using, for example, transposition or substitution. For a more detailed experience of the history of cryptography see [2, 1].

## 3.3 The Distribution of Keys

> How's that...the key's run off.
>
> - *Jack Sparrow, Pirates of the Caribbean*

The secure delivery of a key - or key distribution as it is known - is at the heart of what cryptography is essentially about. There are two main types of key distribution: private and public.

### 3.3.1 Private Key Distribution

Private key distribution is where the security of a protocol is dependent on the safe delivery of the key between Alice and Bob. During the Second World War, the German military used the famous Enigma machines. The Enigma was a complex encryption device that relied on the daily distribution of code books (the key) in order to ensure security. Because these keys could only be entrusted to the intended recipient, it is an example of private key distribution. Another famous protocol that relied on the distribution of a private key was the Vernam cipher or the "one-time pad" as it is also known.

**The One-Time Pad**

The one-time pad was developed in 1917 by an American engineer Gilbert Vernam, and was significantly used during the latter stages of World War 1. It is the only known classical cypher that is proven to be 100% secure [3]. The one-time pad cypher, as its name suggests, can only be used once [4] and the randomness is the key to its security. This is because if the key was reused a number of times, a potential code breaker could build up patterns from the rehashed key. We will see later on that the one-time pad is also an important tool used in quantum cryptography. Figure 3.2 gives a simple illustration of

[3] Its complete security was actually only proved 10 years after World War 1 by Charles Shannon.

[4] The word pad comes from the fact that spies used to quickly write something down on a note book or pad and then tear it off.

the one-time pad. Alice and Bob agree on a random key, 10010111, before hand and then go their separate ways. They can then encrypt a message using the key and be completely confident that their message will not be broken. As we will see in subsequent chapters, quantum cryptography is an example of private key distribution.

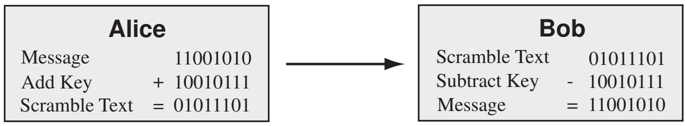


**Figure 3.2:** The One-Time Pad is the only known classical cryptography protocol that is completely secure. Here the key is added by Alice and subtracted by Bob, using modulus 2.

### 3.3.2 Public Key Distribution

Up until now all cryptographic protocols consisted of people having to get together to agree on a suitable key before hand and to then privately deliver the secure key. But in the 1970s, a new way of sending the key was conceived: public key distribution. In short, public key distribution is where everyone knows what the key is. Therefore Alice and Bob do not need to agree on the secret key before hand and there is no need to ensure the key is kept secure. The best thing about this idea is it eliminates the need for Alice and Bob to meet at all. Public key distribution originated in the early 1970s with an idea by James Ellis and a subsequent application by Clifford Cocks. But due to national security, as Ellis and Cocks worked for British Intelligence, their ideas were not known to the general public until many years later. So it was initially thought that public key distribution was discovered by the academic community in 1976 by Whitfield Diffie, Martin Hellman and Ralph C. Merkle at Standford University. In 1978, Ronald Rivest, Adi Shamir, and Leonard Adleman, developed the first practical implementation of public key distribution, known as RSA (what Cocks demonstrated a few years earlier).

Public key distribution protocols work on a simple mathematical principle: one way functions. These are mathematical operations that are easy enough to go from A to B but are very difficult going from B to A. An analogy of this would be the common padlock. Bob sends Alice an open padlock. Alice then uses the padlock to securely lock say a box of secret material. Alice then sends this to Bob who can open the padlock as it was his padlock to start with. So in this case the padlock is the key and it is distributed publicly. The RSA protocol works in a similar way and is based on the factorization of large prime numbers. For example it is quick and easy to work out that $211 \times 23 = 4853$. However it is a lot slower and more difficult to find that the prime factors of 4853 are 211 and 23. There is no known algorithm that factorizes large prime numbers in a short period of time. So by the time the prime number has been factorized, it has been replaced with a new large prime number.

## 3.4 Shor's Algorithm

A quantum computer is a device that uses properties of quantum mechanics to do a number of calculations simultaneously. In 1994 Peter Shor theoretically showed that a quantum computer would be able to factorize a large number exponentially faster than a classical computer - this became known as Shor's algorithm [23]. The possible advent of quantum computers would result in current encryption programs like RSA to be broken almost immediately. Unlike private key distribution, where the one-time pad is 100% secure, there are no known perfectly secure public key protocols. The RSA code can be broken, but it takes a classical computer, say 3 months to break; but a quantum computer has the potential to break it in a matter of seconds or minutes. However, even with the introduction of quantum computers, the one-time pad is still completely secure. This is an important fact, because as we will see later, the one-time pad is used in the final stage of quantum cryptographic protocols. The reason that the one-time pad is still secure, is because it uses a randomly generated key. So a quantum computer would generate a number of possible random keys but it would not know which of them is the correct one.

Quantum theory is a double-sided sword. It has the potential to make current encryption algorithms obsolete. However, it is also responsible for the next evolution of code-makers: quantum cryptographers.

## 3.5 Conclusion

We have discussed aspects of cryptography and how it is used today in the protection of important information. We finished off with mentioning how quantum theory will enable us, via quantum computers, to break current public key distribution protocols such as RSA, thus giving the code-makers an edge. However, as we will soon see, quantum theory not only weakens current ciphers but gives us a more exciting way of encrypting information through quantum key distribution.

Chapter 4

# Quantum Cryptography

> I'm telling secrets to the one guy you don't tell secrets to.
>
> - *Russell Hammond, Almost Famous*

This chapter introduces quantum cryptography and begins with its origins in quantum money. It then discusses the two main forms of quantum cryptography: discrete and continuous. Key distillation and privacy amplification are then introduced with a final comment on the experimental versions of quantum cryptography.

## 4.1 Introduction

So what is wrong with classical cryptography? Well, as we have seen from the end of the last chapter, the security of classical cryptography is compromised with the possible advent of a quantum computer. Historically, whenever the code-makers think they have developed a new encryption protocol, the code-breakers come along and break it. So even though quantum theory can be used by both code-makers and code-breakers, the struggle might well be over. Code-makers have a new technique available that has the capacity to encrypt secrets forever. This technique is known as quantum cryptography.

## 4.2 Quantum Money

> (Quantum Money) Can't Buy Me Love.
>
> - *The Quantum Beatles*
>
> Note: The Quantum Beatles, although not as famous as the Fab Four, still had a number of hits including, You Never Give Me Your Quantum Money and Quantum Money (That's What I Probabilistically Want).

Quantum cryptography started from an unusual idea formed in 1969 by graduate student Steven Wiener. He decided to use properties of quantum mechanics to create bank notes that were unable to be counterfeited. These bank notes where known as quantum money. Each note consisted of a number of "light traps" (unseen from view) that held a random combination of one of four possible photon polarizations: $\updownarrow$ (90 degrees) $\leftrightarrow$ (0 degrees) ↗ (45 degrees) and ↘ (135 degrees). As well as having these "light traps", each bank note had a corresponding serial number. The bank would keep a list of all bank notes and their serial numbers along with their respective polarization combinations. For example, Fig. 4.1 (a) the bank would record that the bank note with serial number 4444 has the polarization sequence ↔ ↕ ↕ ↗ ↗ ↘ ↔ ↗. Now the counterfeiter (known as Charlie) can easily read and therefore copy the serial number. However the reading (or measuring) of the polarizations is a different manner. Now Charlie can measure the polarizations using one of the four filters (or bases): ↕ ↔ ↗ ↘. If he measures a photon polarized ↕ using the ↕ basis then he has made the correct measurement and can therefore correctly identify how the photon was initially polarized. Conversely if the photon was originally polarized ↔ and he used the same basis then because it is orthogonal to the basis no light will be let in and he will know how it was originally polarized. However if the photon was initially↗ or↘ and Charlie used the ↕ or ↔ basis he runs into a problem. This problem is due to quantum mechanics. What happens is that if ↕ is used to measure, say ↗, then there is a 50% chance a 45 degree polarization could "rotate" down to the horizontal or a 50% chance it will "rotate" up to the vertical. In the chance that the latter happens then Charlie has correctly measured the polarization of the photon. However if the former happens then he will incorrectly assumed that the initial polarization was in fact ↔. Figure 4.1 (b) shows one attempt by a counterfeiter to replicate the banks original note, where only two of the eight polarizations are correct. It is in this way that quantum mechanics provides 100% security to bank notes! Now quantum money is completely impractical. The cost of making this happen is a lot more than the cost of a particular bank note and the embedding of the light polarizations for long periods of time is also not possible. But it was the unexpected starting point for quantum cryptography.

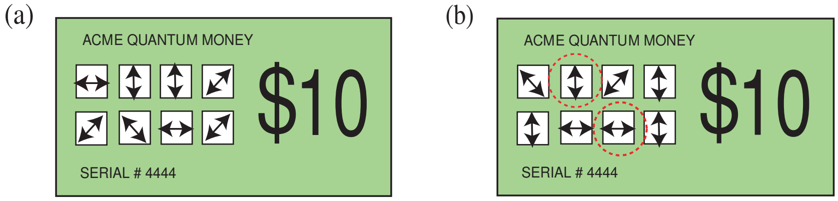


**Figure 4.1:** Example of quantum money. Each bank note had a serial number and random polarizations (which, although shown here, would not be viewed). (a) The original bank note (b) The counterfeiter's bank note, with the circles indicating the right measurements. The only difference between the two cannot be observed! Quantum mechanics enables the total security against forgery. This idea was thought of by Steven Wiesner in 1969 and lead to the first development of quantum cryptography.

### 4.2.1 The No Cloning Theorem

Before we go into quantum cryptography you might ask why don't we just copy or clone the particular polarizations and then we can create a perfect counterfeit? This brings us to the non cloning theorem

*An unknown quantum state cannot be cloned*

An important consequence of this theorem[1] is it stops Charlie from simply copying the specific polarizations of the bank note. This is because they are unknown states, i.e. Charlie did not prepare them himself. So the no cloning theorem does not stop Charlie from copying it all, but it forbids him from perfectly copying it. A short proof of the no cloning theorem follows. We have $\mathrm{U}(|\alpha\rangle|0\rangle) = |\alpha\rangle|\alpha\rangle$ [2] along with $\mathrm{U}(|\beta\rangle|0\rangle) = |\beta\rangle|\beta\rangle$ with $|\beta\rangle \neq |\alpha\rangle$. Now if we take the inner product we have $\langle\alpha|\beta\rangle = \langle\alpha|\langle\alpha|\beta\rangle|\beta\rangle$, which is equal to $\langle\alpha|\beta\rangle^2 = \langle\alpha|\beta\rangle$. This equation has either 1 or 0 as the solutions. So if it is 1 then $\alpha$ and $\beta$ are equal and are the same state. And if it is equal to 0, then $\alpha$ and $\beta$ are orthogonal and are therefore distinguishable. Therefore we can only copy states that are orthogonal to each other but there does not exist a universal cloning machine that is able to copy all nonorthogonal arbitrary states. This is an important property that is inherent in any eavesdropping attack in quantum cryptography.

## 4.3 Discrete Quantum Cryptography

After discussing Wiesner's idea with him, Charles Bennett, a scientist from Bell Labs in New York, continuously thought of his idea over the next few years. This lead to Bennett and another scientist, Gilles Brassard, to come up with quantum cryptography [16]. This form of cryptography, know known as discrete quantum cryptography [17], deals with the building and distribution of the key rather than the secure transmission of a message. Therefore quantum cryptography is also known as quantum key distribution. Once quantum cryptography is used to generate a secure key, the one time pad (see Chapter 3) from classical cryptography, is used to encrypt the message. This only works because the key is generated from a random key - which is a requirement of the one time pad. This discrete quantum cryptographic protocol became known as the BB84 protocol, after the authors (Bennett and Brassard) and the year it was first published (1984).

### 4.3.1 The BB84 Protocol

Alice and Bob have at their disposal four possible photon polarizations ↕ ↔ ⤢ ⤡. They then allocate a bit value of 1 to the ↕ ⤢ polarizations and a bit value of 0 to the ↔ ⤡ polarizations. From these four polarizations there are two possible orthogonal bases: +(or rectilinear) basis formed from the ↔ and ↕ polarizations and the ×(or diagonal) basis formed from the ⤡ and ⤢ polarizations. The BB84 protocol goes as follows

[1] Even though one can clone a sheep, one cannot clone a single photon! - *Andrew Steane*

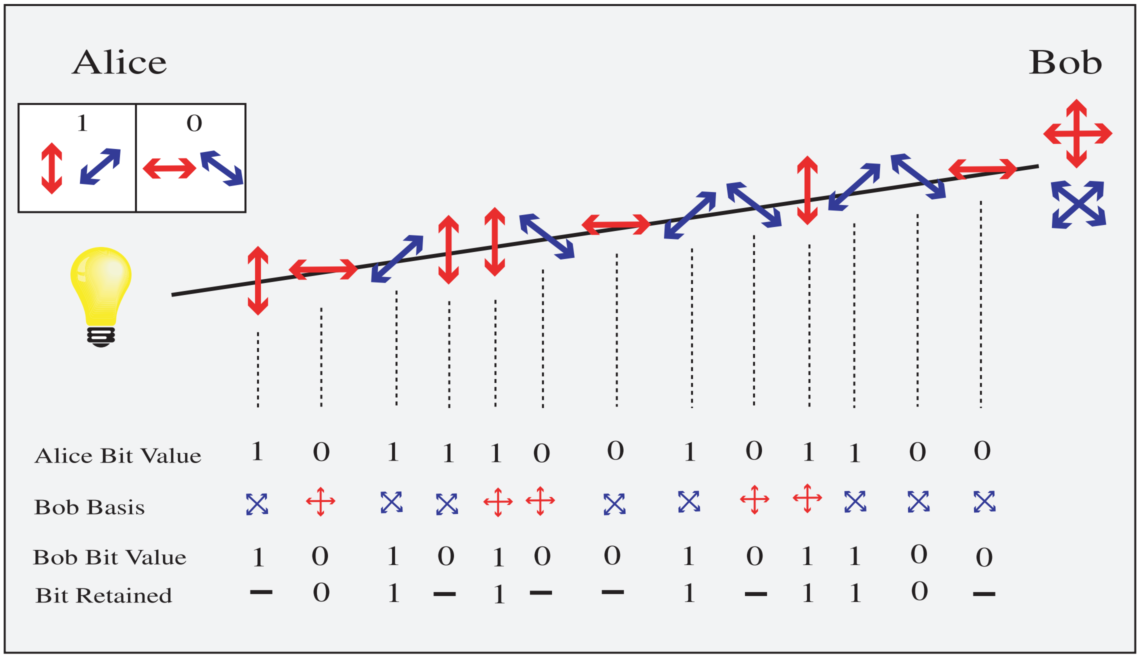
[2] U here is known in quantum mechanics as the unitary operator. There is no need to go into detail about this operator except to say that in our case its function is to evolve a ket in time.

1. Alice sends a random ensemble chosen from the four polarizations ↔, ↕ ⤢ ⤡.
2. Bob measures the ensemble Alice has sent by randomly switching between the + × bases.
3. Alice tells Bob (over a classically insecure channel) at which times he measured the correct bases.
4. Alice and Bob discard the times when they did not use the same bases.
5. Alice and Bob then test the security of their key by using a randomly chosen subset of their key. Results of their subset are compared and if errors are detected, the transmission is insecure and they abort and start again.
6. Classical key distillation and privacy amplification techniques are used to generate a secure key.
7. The one time pad is used to encrypt a message.

Figure 4.2 gives an example of the BB84 protocol. In the first step Alice sends the following random polarizations ↕ ↔ ⤢ ↕ ↕ ⤡ ↔ ⤢ ⤡ ↕ ⤢ ⤡ ↔ which has the following bit value sequence 1011100101100. In the second step Bob receives this ensemble and measures the photons with the following randomly chosen bases × + × × + + × × + + × × ×. These measurement bases result in Bob getting the following bit sequence 1010100101100. The next step involves Bob telling Alice the particular measurement bases he chose at each time interval and then Alice telling him when he chose the correct basis. They then discard the times when they did not agree on the same basis and keep only the correct bases. It is from this that Alice's original key is reduced and they end up with a correlated key.

### Eavesdropping the BB84

Eve is in the same position as Bob when it comes to correctly guessing what specific polarizations Alice has sent. So it might be thought that if she is in the same position, then she can do just as well. But she is effectively shut out of the picture by the classical communication. It works like this. Suppose Eve randomly measures what Alice has sent and she records what she thinks is the correct results. She then sends these to Bob who also performs his measurements on them. But Bob is now measuring what Eve has sent and not what Alice originally sent. Due to the uncertainty in the measuring of photon polarizations Eve and Alice's data would almost certainly be completely different. So as we can see, she is unable to effectively manipulate the data once Bob has his or alternatively if she does manipulate the data initially, she introduces too many errors into the transmission, which Alice and Bob would detect. Also, as we have previously discussed, Eve is unable to simply perfectly copy the states due to the no cloning theorem.



| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Alice Bit Value | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 |
| Bob Basis | × | + | × | × | + | + | × | × | + | + | × | × | × |
| Bob Bit Value | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 |
| Bit Retained | – | 0 | 1 | – | 1 | – | – | 1 | – | 1 | 1 | 0 | – |

**Figure 4.2**: Discrete quantum cryptography or BB84 protocol

### Other Discrete Protocols

There have been a number of variations of the discrete protocol since it was first introduced. Bennett himself introduced a new discrete protocol in 1992, known as the B92 protocol [24]. It showed that you could set up a secure transmission between Alice and Bob by using only two nonorthogonal states. So Eve needs to distinguish between two nonorthogonal states, which due to quantum mechanics, is impossible to do without adding any additional errors. However the security proofs for B92 are not yet as strong as for BB84. Another protocol devised by Ekert [25], involved the use of entanglement through the EPR pairs [7] and also solved the key storage problem. Other schemes include: a 6 state quantum cryptographic protocol [26], the use of quantum memory [27], and other variations [28].

## 4.4 Continuous Variable Quantum Cryptography

As an alternative to the discrete polarizations of the BB84 protocol, in 1999 Tim Ralph came up with a continuous version known as continuous variable quantum cryptography [19]. It involves the encoding of information onto continuous variables, such as non-commuting quadratures, in order to establish a secure key between Alice and Bob. Ralph then updated his initial idea the following year to include squeezed states [29]. In the same year, Hillary [30] introduced a variation of this squeezed state protocol that used binary modulated squeezed light. Margaret Reid showed [31] that quantum correlations in the form of EPR pairs could also be used to distill a secret key. Then in 2001, Gottesman and Preskill analyzed the security of continuous variables by giving a proof that showed how quantum key distribution using squeezed states was unconditionally secure [32]. Also in 2001, Cerf et al [33] devised an improved squeezed state protocol that relied on the sending and detecting of gaussian information. In 2002, Grosshans and Grangier [34] introduced a new continuous variable protocol that used coherent states instead of squeezed states. They showed that coherent states afforded the same degree of security as compared to protocols that used squeezed light and entanglement.

### 4.4.1 Steps of the Protocol

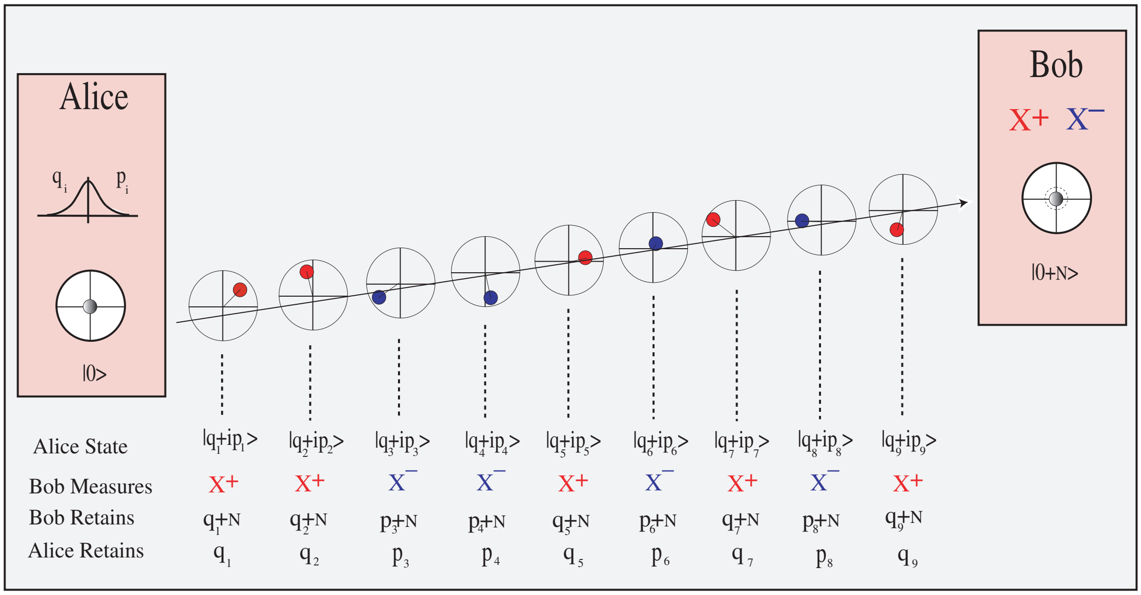
The protocol of the continuous regime is similar to that of the discrete regime and deals with gaussian distributions and gaussian statistics in general. This is an important point, as all analysis used in this thesis, use formulae and equations that are dependent on the fact that we start off with and ultimately measure gaussian distributions. The continuous variable version of our protocol is based on the one developed by Grosshans and Grangier [34] and goes as follows:

1. Alice draws two real random numbers q and p from a gaussian distribution.
2. She displaces a vacuum state $|0\rangle$ by p and q to form a new random state $|q + ip\rangle$.
3. Alice repeats steps (1) and (2) a number of times, sending the states to Bob.
4. Bob randomly switches between the measurement bases Q and P.
5. Bob tells Alice, over an insecure classical line, what basis he chose to measure at each particular time interval.
6. Classical key distillation and privacy amplification techniques are used to generate a secure key.
7. Alice and Bob then test the security of their key by using a randomly chosen subset of their key. Results of their subset are compared and if errors are detected, the transmission is insecure and they abort and start again.
8. The one time pad is used to encrypt a message.

The allocation of bits for the continuous variable system is achieved during the key distillation section (see Section 4.5). Two particular continuous variable quantum cryptography protocols are direct reconciliation and reverse reconciliation.

### 4.4.2 Direct and Reverse Reconciliation Protocols

The protocol above is an example of direct reconciliation as introduced by Grosshans and Grangier [35]. This is where Bob has to correct his data to agree with what Alice has sent. So the classical communication is in the same direction as the initial quantum communication. Now direct reconciliation, and all previous protocols up until that point, were thought to be secure for only line transmissions greater than 50% or 3dB line loss. The first protocol that was shown to beat the 3dB loss limit was using postselection [36]. This is where Alice and Bob post select a subset of their data that they known is entirely secure from Eve. Postselection is durable under high losses and is discussed in more detail in Chapter 6. Reverse reconciliation was theoretically revealed in 2002 [35] and



| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Alice State | $\|q_1+ip_1>$ | $\|q_2+ip_2>$ | $\|q_3+ip_3>$ | $\|q_4+ip_4>$ | $\|q_5+ip_5>$ | $\|q_6+ip_6>$ | $\|q_7+ip_7>$ | $\|q_8+ip_8>$ | $\|q_9+ip_9>$ |
| Bob Measures | X+ | X+ | X− | X− | X+ | X− | X+ | X− | X+ |
| Bob Retains | $q_1+N$ | $q_2+N$ | $p_3+N$ | $p_4+N$ | $q_5+N$ | $p_6+N$ | $q_7+N$ | $p_8+N$ | $q_9+N$ |
| Alice Retains | $q_1$ | $q_2$ | $p_3$ | $p_4$ | $q_5$ | $p_6$ | $q_7$ | $p_8$ | $q_9$ |

**Figure 4.3**: The continuous variable quantum cryptography protocol

experimentally demonstrated in 2003 [37]. It was in contrast to direct reconciliation as it was able to beat the 3dB loss limit. It consisted of Alice trying to correct her data to agree with what Bob had ultimately measured. So the flow of classical information was in the reverse or opposite direction to the initial quantum communication.

### 4.4.3 Eavesdropping

There are a few different attacks that Eve could do against a continuous variable quantum key distribution protocol. These can be categorized as either a direct attack or a passive attack. In the direct attack (or intercept and resend attack) Eve would tap the quantum channel and then gather the information. She would then manipulate the data, without revealing herself, by which time she would then send it onto Bob. A passive attack is when Eve obtains the data, stores it in quantum memory, and then sends it onto Bob. However in this case, unlike a direct attack, she in unable to manipulate the data once Bob has measured it. There are also specific attacks for particular protocols. For example, the reverse reconciliation protocol, an entangling cloner attack has shown to be the optimal attack [35]. This particular attack can be thought of as Eve creating two quantum correlated states, one that she keeps and the other she sends to Bob. In the direct reconciliation (and postselection) protocol, a direct attack using a beam splitter is used as Eve's attack [34, 36].

## 4.5 Key Distillation and Privacy Amplification

> I like my privacy.
>
> - *Shrek*

This section will cover some concepts that are used once a raw key has been exchanged. These are no longer quantum mechanical techniques but are purely classical in nature. These include the correcting of errors between Alice and Bob (key distillation) and reducing Eve's knowledge of the final key (privacy amplification).

### Key Distillation

The main point of key distillation is to produce a key between Alice and Bob which has negligible errors. Eve also has the key but with many more errors. The two key distillation techniques that will be used or are mentioned in this thesis, include reverse reconciliation (see Chapter 5) and postselection (see Chapter 6). The allocation of bits in the BB84 protocol is agreed upon before Alice sends her ensemble. In the continuous variable version, the allocation of bits occurs after Bob has recorded his results. One example of this is the "sliced reconciliation" protocol [38]. This process can be thought of as the extraction of a binary key from a gaussian distribution. The slice reconciliation protocol gives up to 95% of the Shannon's limit (see Chapter 5). Figure 4.4 gives an illustration of the sliced reconciliation protocol. Alice has a gaussian distribution (red/thin line) from the data that she has sent and Bob has the corresponding gaussian distribution (blue/thick line). This is the same as Alice's data, except that it has noise added on and so is a wider distribution. Alice and Bob then divide their data into n slices[3]. Therefore there would be n slices, with n+1 regions or bins, and n-1 bits/symbol. Examples with n=3 and n=7 are given in Fig. 4.4.

### Privacy Amplification

Privacy amplification, as its name suggests, is a classical process where Alice and Bob reduce Eve's knowledge of the key to zero and therefore maintain their privacy. Privacy amplification was first reported in 1988 by Bennett et al. [39,40]. This was then extended a few years later and has also lead to interest from the classical cryptography community [41].

## 4.6 Experimental Quantum Cryptography

> There was never any doubt that it would work, only that our fingers would be too clumsy to build it.
>
> - *Charles Bennett*

Charles Bennett was continually asked about whether the idea that he and Gilles Brassard came up with, could be put into practice. For years he would always reply that "there

[3] These slices in practice would not be equal in length as they are in Fig. 4.4. Each region or bin, as it is known, would need to contain equal areas underneath the graph. This is so each bit of data is weighted equally.

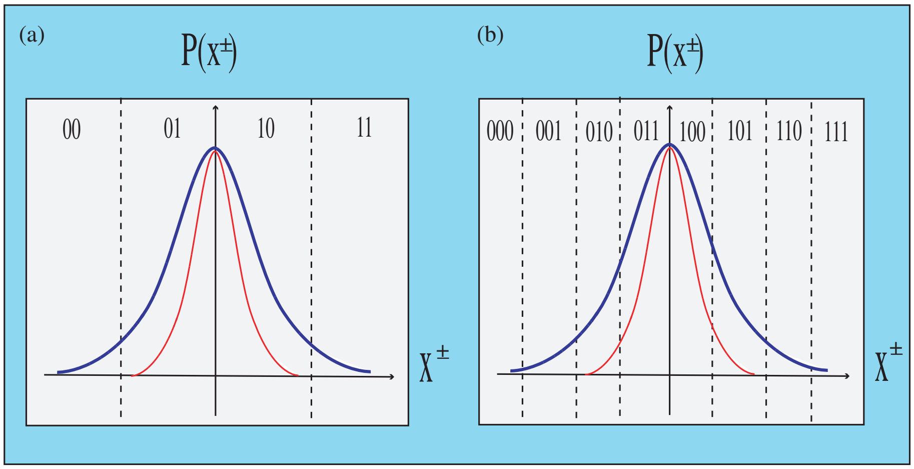


**Figure 4.4:** Sliced reconciliation is a protocol that extracts a bit sequence from gaussian information. Red is Alice's gaussian distribution and blue is Bob's. (a) Using 3 slices, 2 bits/symbol (b) Using 7 slices, 3 bits/symbol.

was no need to go to the north pole if you know it exists". However this reasoning did not last long. Bennett and others decided to do the first experimental demonstration of quantum cryptography, and in 1989 achieved their goal [42]. The experiment was performed at IBM in New York, where Alice and Bob were separated by a distance of only 30cm! Other discrete quantum key distribution experiments performed since, have predominately switch between the BB84 and B92 protocols [43, 44, 45]. Some of these experiments, instead of using polarization encoding, used phase encoding in the allocation of bits. Ekert's scheme of using EPR pairs [25] was also experimentally demonstrated by three groups in 2000 [46,47,48]. Because of its late start, continuous variable quantum key distribution has only had a fraction of the experiments that the discrete variable has. The first coherent state continuous variable quantum cryptographic experiment was performed in 2003 by Grosshans et al [37]. The commercial potential of a reliable and working quantum key distribution scheme is unbounded. The commercial companies utilizing today's classical encryption protocols, such as RSA, are worth hundreds of millions of dollars. It is only a question of when quantum cryptography catches up to that level.

## 4.7 Conclusion

This chapter introduced the concept of quantum money and how it was the initial inspiration for quantum cryptography. The two main forms of quantum cryptography, discrete and continuous, were discussed, along with the various steps in each of their protocols. The importance of key distillation and privacy amplification, in distilling a secure key, was also highlighted. Finally it ended with the experimental forms of quantum cryptography.

Chapter 5

# Quantum Cryptography Without Basis Switching

> With quantum states, what we achieve
> Defeats whatever you conceive.
> So even Bob has to believe
> That you can't hear us, can you Eve?"
>
> \- *John Preskill*

This chapter investigates a new coherent state quantum key distribution protocol that eliminates the need to randomly switch between measurement bases. This protocol provides significantly higher secret key rates with increased bandwidths than previous schemes that only make single quadrature measurements. It also offers the further advantage of simplicity compared to all previous protocols which, to date, have relied on switching.

The work presented here has been published in the journal article:

- Christian Weedbrook, Andrew Lance, Warwick Bowen, Thomas Symul, Tim Ralph and Ping Koy Lam. *Quantum Cryptography Without Switching*, Physical Review Letters **93**, 170504 (2004).

## 5.1 Introduction

As we have seen from previous chapters, quantum cryptography is the science of sending secret messages via a quantum channel. It uses properties of quantum mechanics [17, 18] to establish a secure key, a process known as quantum key distribution [16]. This key can then be used to send encrypted information. In a generic quantum key distribution protocol, a sender (Alice) prepares quantum states which are sent to a receiver (Bob) through a potentially noisy channel. Alice and Bob agree on a set of non-commuting bases to measure the states with. Using various reconciliation [49, 50] and privacy amplification procedures [9], the results of measurements in these bases are used to construct a secret key, known only to Alice and Bob. Switching randomly between a pair of non-commuting measurement bases ensures security: in a direct attack, an eavesdropper (Eve) will only choose the correct basis half the time; alternatively, if Eve uses quantum memory and performs her measurements after Bob declares his basis, she is unable to manipulate what Bob measures. It is commonly assumed that randomly switching between measurement bases is crucial to the success of quantum key distribution protocols. In this chapter we show that this is not the case, and in fact greater secret key rates can be achieved by simultaneously measuring both bases.

The original quantum key distribution schemes in the discrete variable regime were based on the transmission and measurement of random polarizations of single photon states [17]. Other discrete variable quantum key distribution protocols have been proposed [25] and experimentally demonstrated [46] using Bell states. However the bandwidth of such schemes is experimentally limited by single photon generation and detection techniques. Consequently in the last few years there has been considerable interest in the field of continuous variable quantum cryptography [51], which provides an alternative to the discrete approach and promises higher key rates. Continuous variable quantum key distribution protocols have been proposed for squeezed and Einstein-Podolsky-Rosen entangled states (see Chapter 4). However, these protocols require significant quantum resources and are susceptible to decoherence due to losses. Quantum key distribution protocols using coherent states were proposed to overcome these limitations. Originally such schemes were only secure for line losses less than 50% or 3dB [29, 34]. This apparent limitation was overcome using the secret key distillation techniques of post-selection [36] and reverse reconciliation [35].

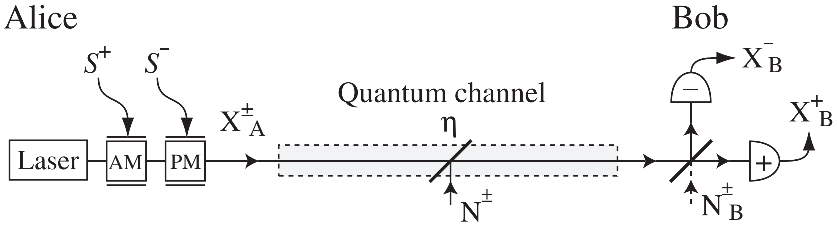


**Figure 5.1:** Schematic of the simultaneous quadrature measurement protocol. $\mathcal{S}^{\pm}$: random Gaussian numbers, AM: amplitude modulator, PM: phase modulator, $\hat{X}_A^{\pm}$: quadratures of Alice's prepared state, $\eta$: channel transmission, $\hat{N}^{\pm}$: channel noise, $\hat{X}_B^{\pm}$: observables that Bob measures and $\hat{N}_B^{\pm}$: Bob's vacuum noise.

In general, security in discrete variable cryptography protocols is ensured via random switching between measurement bases [17] or random switching of state manipulation [52]. The random switching between measurement bases can be achieved simply via a 50/50 beam splitter, where the selection of the measurement basis is chosen through the random photon transmission and reflection statistics. To date all continuous variable cryptography protocols have also relied on randomly switching between non-commuting bases. In the continuous variable regime, switching requires precise control of the phase of a local oscillator beam, which is difficult to achieve in practice. This local oscillator switching currently places a serious technical limitation on the bandwidth of cryptography protocols. In this chapter, we introduce a new coherent state protocol that does not require switching.

## 5.2 The Simultaneous Quadrature Measurement or SQM Protocol

Unlike previous continuous variable quantum key distribution protocols, this protocol measures both bases simultaneously, utilizing the quantum channel more effectively and achieving both higher secret key rates and bandwidths compared to previous continuous variable quantum key distribution protocols.

The quantum states we consider in this chapter can be described using the field annihilation operator $\hat{a}=(\hat{X}^{+}+i\hat{X}^{-})/2$, which is expressed in terms of the amplitude $\hat{X}^{+}$ and phase $\hat{X}^{-}$ quadrature operators. Without a loss of generality, the quadrature operators can be expressed in terms of a steady state and fluctuating component as $\hat{X}^{\pm}=\langle\hat{X}^{\pm}\rangle+\delta\hat{X}^{\pm}$, which have variances of $V^{\pm}=\langle(\delta\hat{X}^{\pm})^{2}\rangle$. Figure 5.1 shows a schematic of our protocol, which we term the simultaneous quadrature measurement protocol, or simply, the SQM protocol. Here the phase and amplitude of a laser is modulated and sent to Bob via a quantum channel with efficiency $\eta$. Bob then measures the incoming states through double homodyne detection with a 50/50 beam splitter (see Chapter 2); where, unlike homodyne detection, there is no switching of the measurement quadratures, i.e. they are measured at the same time.

### 5.2.1 Steps of the SQM Protocol

Our scheme is similar to the continuous variable coherent state quantum cryptography protocols originally presented in [29, 34] and illustrated in Chapter 4. It goes as follows. Alice draws two random real numbers $\mathcal{S}^{+}$ and $\mathcal{S}^{-}$ from Gaussian distributions with zero mean and a variance of $V_{\mathcal{S}}^{\pm}$. She then prepares a state by displacing the amplitude and phase quadratures of a vacuum state by $\mathcal{S}^{+}$ and $\mathcal{S}^{-}$, respectively. The quadrature operators of Alice's state are therefore given by

$$\hat{X}_{\mathrm{A}}^{\pm}=\mathcal{S}^{\pm}+\hat{X}_{\mathrm{v}}^{\pm} \tag{5.1}$$

where $\hat{X}_{\mathrm{v}}^{\pm}$ are the quadrature operators of the initial vacuum state. The resulting state has normalized quadrature variances of $V_{\mathrm{A}}^{\pm}=V_{\mathcal{S}}^{\pm}+1$, i.e. $\langle(\hat{X}_{v}^{\pm})^{2}\rangle=1$ and $\langle\mathcal{S}^{\pm}\hat{X}_{v}^{\pm}\rangle=\langle\hat{X}_{v}^{\pm}\mathcal{S}^{\pm}\rangle=0$. Alice transmits this state to Bob through a quantum channel with channel transmission efficiency $\eta$, which couples in channel noise $\hat{N}^{\pm}$, where the variances of the channel noise must obey the uncertainty relation $V_{N}^{+}V_{N}^{-}\geq 1$. Bob simultaneously measures the amplitude and phase quadratures of the state using a 50/50 beam splitter. The state Bob receives is given by

$$\hat{X}_{B}^{\pm}=\frac{1}{\sqrt{2}}\Big(\sqrt{\eta}\hat{X}_{A}^{\pm}+\sqrt{(1-\eta)}\hat{X}_{N}^{\pm}+\hat{N}_{B}^{\pm}\Big) \tag{5.2}$$

Therefore the quadrature variances of the state measured by Bob are given by

$$V_{B}^{\pm}=\frac{1}{2}\Big(\eta V_{A}^{\pm}+(1-\eta)V_{N}^{\pm}+1\Big) \tag{5.3}$$

where the cross terms are uncorrelated and are therefore zero. Alice and Bob then test the channel transmission and bit error rate of their key by using a randomly chosen subset of their key to check for errors. If the errors are within some tolerated limit, they then use secret key distillation and privacy amplification techniques (see Chapter 4), to distill a common secret key. This protocol is essentially the same as the one presented in Section 4.4.1, except for the important fact that Bob now neglects the switching of measurement bases, in favor of measuring both bases simultaneously. Another step that has been changed is there is no need for Alice and Bob to classically differentiate which quadratures were measured. It is possible to analyze our protocol using either the post-selection (see Chapter 6), or reverse reconciliation (see Chapter 4), secret key distillation techniques. However, for simplicity, we limit our analysis to Grosshans and Grangier's reverse reconciliation protocol. In this protocol, both Alice and Eve, try to infer Bob's measurement results.

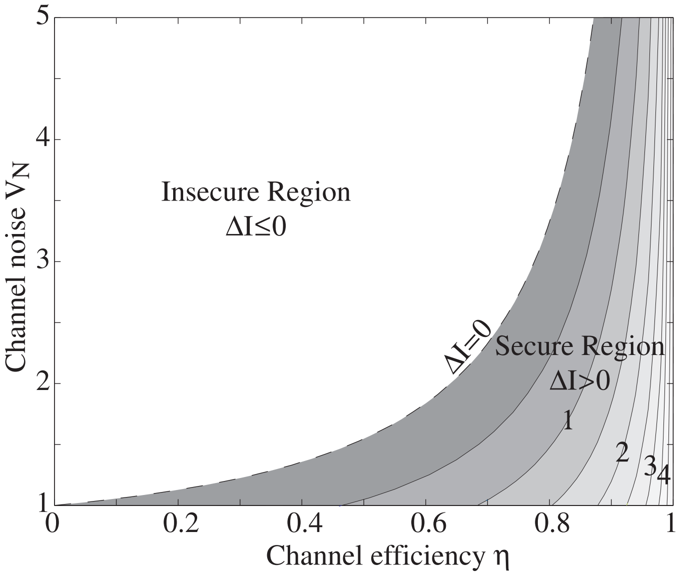


**Figure 5.2:** Contour plot of the information rate for the simultaneous quadrature measurement protocol as a function of channel efficiency $\eta$ and channel noise $V_N$ in units of (bits/symbol) for $V_A = 100$.

### 5.2.2 Conditional Variance

Alice's inference can be characterized by a conditional variance which is used to calculate the secret key rate. This conditional variance is defined as

$$V^{\pm}_{A|B} = \min_{g^{\pm}_{\mathrm{A}}} \langle (\hat{X}^{\pm}_B - g^{\pm}_{\mathrm{A}} \hat{X}^{\pm}_A)^2 \rangle \tag{5.4}$$

and can be thought of as Alice giving her best estimate of what Bob has measured. This is all inherent in the reverse reconciliation process. The gain $g^{\pm}_{\mathrm{A}}$ can be optimized to give a minimum conditional variance. The optimization goes as follows

$$\frac{\partial V^{\pm}_{A|B}}{g^{\pm}_A} = - < \hat{X}^{\pm}_B \hat{X}^{\pm}_A > - < \hat{X}^{\pm}_A \hat{X}^{\pm}_B > + 2 g^{\pm}_A < \hat{X}^{\pm 2}_A > = 0 \tag{5.5}$$

Rearranging for the gain we have

$$g_A^{\pm} = \frac{< \hat{X}_A^{\pm} \hat{X}_B^{\pm} >}{< \hat{X}_A^{\pm 2} >} \tag{5.6}$$

(where we have collected terms based on the fact that $\hat{X}_A^{\pm}$ and $\hat{X}_B^{\pm}$ commute, i.e. $\langle \hat{X}_B^{\pm} \hat{X}_A^{\pm} \rangle = \langle \hat{X}_A^{\pm} \hat{X}_B^{\pm} \rangle$). This gain is then substituted into Eq. (5.4) to give

$$V_{A|B}^{\pm} = V_B^{\pm} - \frac{|\langle \hat{X}_A^{\pm} \hat{X}_B^{\pm} \rangle|^2}{V_A^{\pm}} \tag{5.7}$$

where we denote $V_{A|B}^{\pm}$ as the conditional variance of B given A in the standard form. To calculate a relation between Alice and Eve's conditional variances of Bob's measurement, $V_{E|B}^{\pm}$ and $V_{A|B}^{\pm}$, we define the states that denote Alice's and Eve's inference of Bob's measurement, expressed as:

$$\hat{X}_{E|B}^{\pm} = \hat{X}_B^{\pm} - \alpha \hat{X}_E^{\pm} \tag{5.8}$$

$$\hat{X}_{A|B}^{\mp} = \hat{X}_B^{\mp} - \beta \hat{X}_A^{\mp} \tag{5.9}$$

where $\beta \hat{X}_A^{\pm}$ and $\alpha \hat{X}_E^{\pm}$ are Alice and Eve's optimal estimates with the optimal gains, $\alpha$ and $\beta$ (with $\alpha$ and $\beta \in R$). Finding the commutator of these two equations, and using the fact that different Hilbert spaces commute, we find that

$$[\hat{X}_{E|B}^{+}, \hat{X}_{A|B}^{-}] = [\hat{X}_B^{+}, \hat{X}_B^{-}] = 2i \tag{5.10}$$

This leads to the joint Heisenberg uncertainty relation

$$V_{E|B}^{\pm} \geq \frac{1}{V_{A|B}^{\mp}} \tag{5.11}$$

Therefore, there is a limit to what Alice and Eve can know simultaneously about what Bob has measured. From this inequality it is possible to determine the maximum information Eve can obtain about the state in terms of Alice's conditional variances $V_{A|B}^{\pm}$.

### 5.2.3 Alice's Conditional Variance

To minimize Alice's conditional variance for one of Bob's measurements, Alice can prepare and send squeezed states, instead of coherent states. In this case, the quadrature variance of the states prepared by Alice are given by

$$V_{\mathrm{A}}^{\pm} = V_{\mathcal{S}}^{\pm} + V_{\mathrm{sqz}}^{\pm} \tag{5.12}$$

where $V_{\mathrm{sqz}}^{\pm}$ denotes the quadrature variances of the squeezed state, and we have

$$V_{\mathrm{sqz}}^{\pm} \geq \frac{1}{V_A^{\mp}} \tag{5.13}$$

Substituting Eqs. (5.2, 5.3) into (5.7), we determine Alice's conditional variance to be

$$V_{A|B}^{\pm} = \frac{1}{2}\Big(\eta V_{\mathrm{sqz}}^{\pm} + (1-\eta) V_N^{\pm} + 1\Big) \tag{5.14}$$

### 5.2.4 Eve's Conditional Variance

One might be tempted to use the definition of conditional variance for Eve the same way that we used it for Alice. But the problem here is that we are unable to assume anything about Eve who, unlike Alice, is capable of anything and is only constrained by the laws of physics. So we need to find another way. To find a lower bound on Eve's conditional variances, we first consider her inference of Bob's state prior to the 50/50 beam splitter in his station. As before this is given by $V^{\pm}_{E|B'} = \min_{g^{\pm}_{\mathrm{E}}} \langle(\hat{X}^{\pm}_{B'} - g'^{\pm}_{E}\hat{X}^{\pm}_{E})^2\rangle$, where $'$ labels Bob's state prior to the beam splitter, and $g'^{\pm}_{E}$ is chosen to minimize $V^{\pm}_{E|B'}$. Eve's measurement variance after the beam splitter conditioned on Bob's measurement ($V^{\pm}_{E|B}$) can be expressed in terms of the conditional variance before the beam splitter ($V^{\pm}_{E|B'}$) as

$$\begin{aligned} V^{\pm}_{E|B} &= \left\langle\left(\hat{X}^{\pm}_{B} - g^{\pm}_{E}\hat{X}^{\pm}_{E}\right)^2\right\rangle \\ &= \left\langle\left(\frac{1}{\sqrt{2}}(\hat{X}^{\pm}_{B} + \hat{N}^{\pm}_{B}) - g^{\pm}_{E}\hat{X}^{\pm}_{E}\right)^2\right\rangle \\ &= \frac{1}{2}\left\langle\left(\hat{X}^{\pm}_{B} - \sqrt{2}g^{\pm}_{E}\hat{X}^{\pm}_{E}\right)^2\right\rangle + \frac{1}{2} \\ &= \frac{1}{2}\left\langle\left(\hat{X}^{\pm}_{B} - g'^{\pm}_{E}\hat{X}^{\pm}_{E}\right)^2\right\rangle + \frac{1}{2} \\ &= \frac{1}{2}\left(V^{\pm}_{E|B'} + 1\right) \end{aligned} \tag{5.15}$$

where we have used the fact that Eve has no access to the beam splitter in Bob's station, and therefore has no knowledge of the vacuum entering through it. The minimum conditional variance achievable by Alice, prior to the beam splitter in Bob's station, can be calculated using Eq. (5.7) with $\hat{X}^{\pm}_{B} = \sqrt{\eta}\,\hat{X}^{\pm}_{A} + \sqrt{1-\eta}\,\hat{X}^{\pm}_{N}$, we arrive at

$$V^{\pm}_{\mathrm{A|B'(min)}} = \left(\frac{\eta}{V^{\pm}_{A}} + (1-\eta)V^{\pm}_{N}\right) \tag{5.16}$$

Using the above equation with the conditional variance inequality in Eq. (5.11), we find the value of $V^{\pm}_{E|B'}$. This can then be substituted into the end result of Eq. (5.15) and establish a lower bound on Eve's inferences of Bob's measurements. This bound is given by

$$V^{\pm}_{\mathrm{E|B(min)}} \geq \frac{1}{2}\left(\left(\frac{\eta}{V^{\pm}_{A}} + (1-\eta)V^{\pm}_{N}\right)^{-1} + 1\right). \tag{5.17}$$

### 5.2.5 Secret Key Rate

The rate at which a secure key can be generated, is known as the secret key rate and is given by

$$\Delta I = I_{BA} - I_{BE} \tag{5.18}$$

Therefore $\Delta I$ has to be greater than zero in order to generate a secret key, i.e. Bob and Alice need to share more mutual information than what Bob and Eve does. It must be noted, that it is still possible to generate a secure key if Bob and Eve's mutual information is greater than Bob and Alice's ($\Delta I < 0$), using classical techniques such as advantage

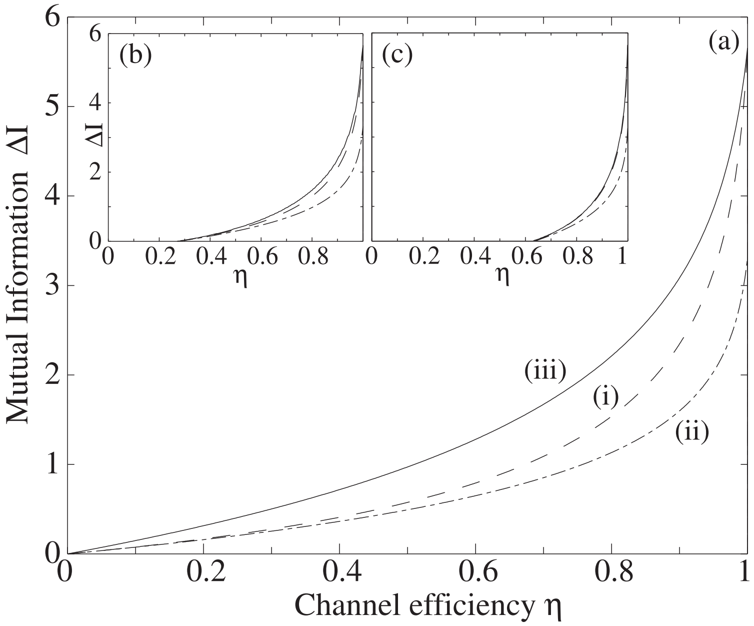


**Figure 5.3:** Net information rates for the simultaneous and single quadrature measurement schemes as a function of channel efficiency. (i) Dashed line, simultaneous quadrature measurement. (ii) Dot dashed line, single quadrature measurement. (iii) Solid line, simultaneous quadrature measurement with feed forward attack. For a variance of $V_A = 100$ with varying channel noise: (a) V$_N$=1 (b) V$_N$=1.2 and (c) V$_N$=2.

distillation [16]. But for this thesis, we will be aiming to achieve positive secret key rates ($\Delta I > 0$). The optimal information rate at which a Gaussian signal can be transmitted though a channel, with additive Gaussian noise, is given by the Shannon formula [53], which can be expressed as

$$I = \frac{1}{2}\log_2(1 + S/N) \tag{5.19}$$

with units of (bits/symbol), where $S/N$ is the standard signal to noise ratio. This optimal net information rate can be used to determine the secret key rate for our simultaneous quadrature measurement protocol. The information rate for the SQM protocol is the sum of the individual information rates for each quadrature. This is given by

$$\begin{aligned}\Delta I &= \Delta I^+ + \Delta I^- \\ &= (I^+_{BA} - I^+_{BE}) + (I^-_{BA} - I^-_{BE})\end{aligned} \tag{5.20}$$

where $I^\pm_{BA}$ is the information rate between Bob and Alice and $I^\pm_{BE}$ is the information rate between Bob and Eve, for both quadratures. These information rates can now be determined from Eq. (5.19). The variance of Bob's measurement $V^\pm_B$ is the signal plus noise, where the noise is Alice (and Eve's) conditional variances $V^\pm_{A(E)|B}$. Therefore we have

$$I^\pm_{BA} = \frac{1}{2}\log_2(\frac{V^\pm_B}{V^\pm_{A|B}}) \tag{5.21}$$

$$I^{\pm}_{BE} = \frac{1}{2}\log_2(\frac{V^{\pm}_{B}}{V^{\pm}_{E|B}}) \tag{5.22}$$

Substituting these expressions into Eq. (5.20), the secret key rate for the simultaneous quadrature measurement protocol can be expressed as

$$\Delta\mathrm{I} \quad = \quad \frac{1}{2}\log_2\Big(\frac{V^{+}_{E|B}V^{-}_{E|B}}{V^{+}_{A|B}V^{-}_{A|B}}\Big) \tag{5.23}$$

Substituting Eq. (5.14) with $V^{\pm}_{\mathrm{sqz}} = 1$ (Alice maximizes her information rate by using coherent states), and Eq. (5.17) into Eq. (5.23), gives a lower bound on the secret key rate

$$\Delta\mathrm{I} \quad \geq \quad \log_2\Big(\frac{(\frac{\eta}{V_A} + (1-\eta)V_N)^{-1} + 1}{\eta + (1-\eta)V_N + 1}\Big) \tag{5.24}$$

where we have assumed symmetry between the amplitude and phase quadratures. Figure 5.2 shows the secret key rate for the simultaneous quadrature measurement scheme as a function of channel efficiency and channel noise. We see that, so long as the channel noise $V_N$ is not excessive, a secret key can be successfully generated between Alice and Bob, even in the limit of very small channel efficiency $\eta$. As the channel noise is reduced, or efficiency increased, the rate at which a key can be established is enhanced. The different shadings reveal the increase in information rate as the channel efficiency is increased. This tends towards an information rate of just over 5.5 bits/symbol for perfect efficiency and a coherent vacuum noise. Figure 5.3 compares the information rates of the SQM protocol (dashed line) and single quadrature (dot dashed line) protocol, as a function of channel efficiency. The two insets give the same comparison but for varying channel noises. In each case, the information rate for the SQM protocol is always higher than that for single quadrature measurement protocol. Also if Alice prepares her states with a large variance (i.e. $V_A = 100$) and with a high channel efficiency, the information rate for the SQM protocol is two times that of the previous best switching scheme. Both schemes collapse (i.e. become less secure) quickly when the channel noise is only slightly increased. This is because we are using reverse reconciliation, which is unable to tolerate high loss. The other solid line in this graph is a possible eavesdropping attack, which is discussed in the next section. The individual secret key rates (i.e. $I_{BA}$ and $I_{BE}$) for the simultaneous and single quadrature measurement protocols can be calculated and are plotted in Fig. 5.5. The shadings in each case, is the area between the individual information rates. Taking the difference at each point gives the net information rate. Therefore a larger shaded area indicates a larger information rate. Thus the SQM protocol clearly dominates the scheme that uses switching in its protocol. It should be noted that in our protocol Eve must attempt to determine Bob's measurements in both the amplitude and phase quadratures at the same time. This introduces an extra penalty to Eve, which is not included in the lower bound for her conditional variance in Eq. (5.17). Therefore, in general, Eve will do even worse than our analysis suggests. The dashed line on the graph is a possible eavesdropping attack that is discussed next.

## 5.3 Eavesdropping Attack

> Confound it all, Samwise Gamgee. Have you been eavesdropping?
>
> - *Gandalf, Fellowship of the Ring*

To establish an upper bound on the secret key rate, we now consider the physical implementation of an eavesdropping attack for Eve for our protocol. In the case where Bob measures a single quadrature, Grosshans and Grangier showed that an entangling cloner attack is the optimum attack [35]. For the SQM protocol, we considered a number of possible attacks for Eve, including various attacks where she injects entanglement into the quantum channel, but found the most effective approach to be a simple feed forward attack with no entanglement as shown in Fig. 5.4.

### 5.3.1 Feed Forward Protocol

The attack goes as follows: Eve taps off a fraction of the beam using a beam splitter with transmission $\epsilon$. She performs simultaneous quadrature measurements on this beam, with measured quadratures of

$$\hat{X}_E^{\pm} = \frac{1}{\sqrt{2}}\Big(\sqrt{1-\epsilon}\,\hat{X}_A^{\pm} + \sqrt{\epsilon}\,\hat{N}_{E1}^{\pm} + \hat{N}_{E2}^{\pm}\Big) \tag{5.25}$$

Therefore the quadrature variances of these states are

$$V_{\mathrm{E}}^{\pm} = \frac{1}{2}\Big((1-\epsilon)V_{\mathrm{A}}^{\pm} + \epsilon + 1\Big) \tag{5.26}$$

where we have normalized the two vacuums entering her beam splitters to one. She then applies the measured photocurrents back onto the quantum channel using electronic feed forward techniques with some gain $g_{\mathrm{E}}^{\pm}$. Now because Eve has reduced the transmission of the quantum channel, she will need to add additional Gaussian noise $\mathcal{N}^{\pm}$ to remain undetected. So after Bob has measured both quadratures simultaneously, he will have

$$\hat{X}_B^{\pm} = \frac{1}{\sqrt{2}}\Big(g_E^{\pm}\hat{X}_E^{\pm} + \mathcal{N}^{\pm} + \sqrt{\epsilon}\hat{X}_A^{\pm} + \sqrt{1-\epsilon}\hat{N}_{E1}^{\pm} + \hat{N}_B^{\pm}\Big) \tag{5.27}$$

where $\hat{X}_E^{\pm}$ is given in Eq (5.25). The variances of Bob's measurements can then be expressed as

$$\begin{aligned} V_{\mathrm{B}}^{\pm} &= \frac{1}{2}\Big(\Big(\sqrt{\epsilon} + g_{\mathrm{E}}^{\pm}\sqrt{(1-\epsilon)/2}\Big)^2 V_{\mathrm{A}}^{\pm} + 1 \\ &+ \mathrm{V}_{\mathcal{N}}^{\pm} + g_{\mathrm{E}}^{2\pm}/2 + \Big(\sqrt{1-\epsilon} + g_{\mathrm{E}}^{\pm}\sqrt{\epsilon/2}\Big)^2\Big) \end{aligned} \tag{5.28}$$

The gain of Eve's feed forward must be chosen carefully to ensure that the magnitude of the signal detected by Bob remains invariant. So for an arbitrary channel transmission $\eta$, Bob would be expecting a signal of $\sqrt{\eta}\,\hat{X}_A^{\pm}$. Now Eve knows that she is sending Bob a signal of $\sqrt{\epsilon}\,\hat{X}_A^{\pm} + g_E^{\pm}\sqrt{(1-\epsilon)/2}\,\hat{X}_A^{\pm}$. Therefore she wants $\sqrt{\eta}\,\hat{X}_A^{\pm} = \sqrt{\epsilon}\,\hat{X}_A^{\pm} + g_E^{\pm}\sqrt{(1-\epsilon)/2}\,\hat{X}_A^{\pm}$. This leads to a gain of

$$g_E^{\pm} = \frac{\sqrt{2}(\sqrt{\eta} - \sqrt{\epsilon})}{\sqrt{1-\epsilon}} \tag{5.29}$$

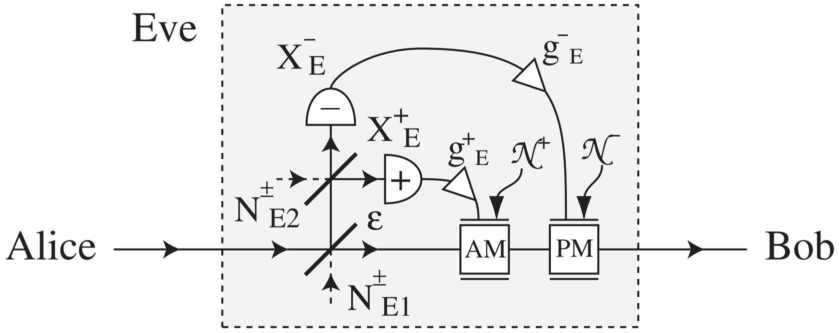


**Figure 5.4:** Schematic of a possible feed forward attack for Eve. $\hat{X}_E^{\pm}$: observables that Eve measures, $\hat{N}_{E1}^{\pm}$ and $\hat{N}_{E2}^{\pm}$: Eve's vacuum noises, $g_E^{\pm}$: Eve's electronic gains and $\mathcal{N}^{\pm}$: additional Gaussian noise.

Substituting this into Eq. (5.28) we obtain Bob's variance due to the feed forward attack of

$$\mathrm{V}_{\mathrm{B}}^{\mathrm{ff}\pm} \;=\; \frac{1}{2}\Big(\eta V_A^{\pm} + \frac{(\sqrt{\eta}-\sqrt{\epsilon})^2}{1-\epsilon} + 1 + \Big(\sqrt{1-\epsilon} + \sqrt{\frac{\epsilon}{1-\epsilon}}(\sqrt{\eta}-\sqrt{\epsilon})\Big)^2\Big) \tag{5.30}$$

## 5.3.2 Conditional Variances and Information Rates

We can now calculate Eve's conditional variance $V_{E|B}^{\mathrm{ff}\pm}$, for the feed forward attack, as a function of the beam splitter transmission $\epsilon$. Deriving Eve's conditional variance as we did for Eq. (5.7) we have

$$V_{E|B}^{\pm} = V_B^{\pm} - \frac{|\langle \hat{X}_E^{\pm}\hat{X}_B^{\pm}\rangle|^2}{V_E^{\pm}} \tag{5.31}$$

Note, unlike last time, we can now use the above equation for Eve. Before we could not as we were unable to assume anything about Eve. But now we are assuming a specific attack and can use the optimize conditional variance formula. Using Eqs. (5.25, 5.27, 5.31) we have

$$\begin{aligned} V_{B|E}^{\pm} \;=\; & \frac{1}{2}((\sqrt{1-\epsilon} - \sqrt{\frac{\epsilon}{1-\epsilon}}(\sqrt{\eta}-\sqrt{\epsilon}))^2 + \eta V_A^{\pm} + \frac{(\sqrt{\eta}-\sqrt{\epsilon})^2}{1-\epsilon} + 1) \\ & - \frac{\Big(\sqrt{\eta(1-\epsilon)} - \sqrt{\epsilon}\Big(\sqrt{1-\epsilon} - \sqrt{\frac{\epsilon}{1-\epsilon}}(\sqrt{\eta}-\sqrt{\epsilon}) + \frac{\sqrt{\eta}-\sqrt{\epsilon}}{\sqrt{1-\epsilon}}\Big)\Big)^2}{2\Big((1-\epsilon)V_A^{\pm} + \epsilon + 1\Big)} \end{aligned} \tag{5.32}$$

Ideally, Eve would take $\epsilon \to 0$ to gain as much information about Alice's signal as possible. However, in doing so she increases the noise on Alice's inference of Bob's state and consequently alerts them to her presence. She must ensure that her attack does not change the magnitude of this noise. This places both lower $\epsilon_{\min}$ and upper $\epsilon_{\max}$ limits on the beam splitter transmission, subject to the constraint that

$$(1-\eta)V_N^{\pm} \;\geq\; \Big(\sqrt{1-\epsilon} - \sqrt{\frac{\epsilon}{1-\epsilon}}(\sqrt{\eta}-\sqrt{\epsilon})\Big)^2 + \frac{(\sqrt{\eta}-\sqrt{\epsilon})^2}{1-\epsilon} \tag{5.33}$$

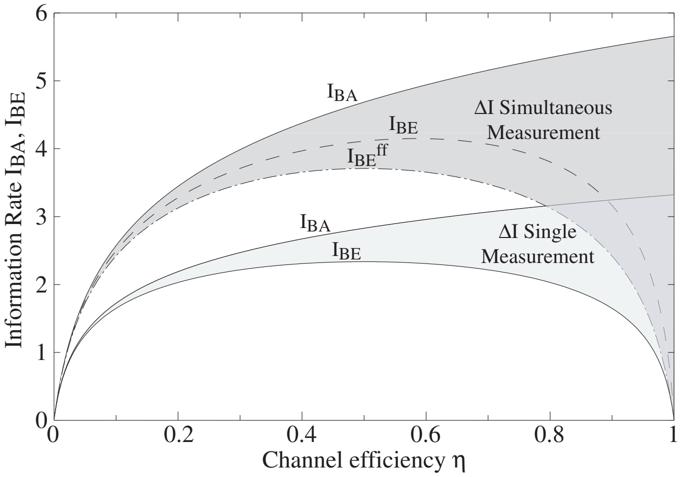


**Figure 5.5:** Information rates for the simultaneous and single quadrature measurement schemes as a function of channel efficiency $\eta$, with $V_A^{\pm} = 100$ and $V_N^{\pm} = 1$. The net information rate for both schemes is $\Delta\mathrm{I} = \mathrm{I}_{\mathrm{BA}} - \mathrm{I}_{\mathrm{BE}}$. In the case of simultaneous quadrature measurements, for $I_{BE}$, the dashed line denotes the hard bound, while the dot dashed line denotes the information rate obtained by Eve using the feed forward attack.

So the left hand side of this inequality is the noise that Bob would have had on his line for an arbitrary transmission $\eta$. This noise must be greater than or equal to the noise that Eve puts on herself; else she would reveal herself. We numerically minimize $V_{E|B}^{\mathrm{ff}\pm}$ for all $\epsilon$ between $\epsilon_{\mathrm{min}}$ and $\epsilon_{\mathrm{max}}$ (see Appendix A). Alice's conditional variance can also be derived using Eq. (5.7) and Eqs. (5.27, 5.28) and is equal to

$$V_{A|B}^{\pm} = \frac{1}{2}\Big((1-\eta)V_N^{\pm} + \eta + 1\Big) \tag{5.34}$$

The above equation is the same as Eq. (5.14). Therefore it is independent of what Eve does because she wants to remain invisible. The secret key rate $\Delta\mathrm{I}^{\mathrm{ff}}$ can now be calculated using Eq. (5.23) and is plotted in Figs. (5.3) and (5.5), and compared with the lower bound calculated in Eq. (5.24). Figure (5.3) shows that for channel noise of variance $V_N = 1$, the feed forward information rate is higher than our lower bound. However, as the channel noise variance is slightly increased (see insets), the feed forward bound asymptotes to the lower bound calculated in Eq. (5.24). In terms of an optimal bound there is a possibility that this attack is the best Eve could do. In Fig. (5.3)(b) and (c), the feed forward attack collapses very quickly with the hard bound we have calculated. This suggests that the feed forward scheme is close to optimal. Also in Fig. (5.3), Eve's information rate is the optimal for the single measurement scheme, represented by a smooth curve. This smooth curve is emulated in the feed forward scheme for the SQM protocol. Again this suggest some correlation with an optimal attack. Of course this needs to be proved and is a possible avenue for future work (see the next Chapter). However, if this is the optimal bound, the benefits of using the SQM protocol are once again seen.

## 5.4 Conclusion

To summarize, we have proposed a new coherent state quantum key distribution protocol based on simultaneous quadrature measurements. We have calculated a lower bound on the secret key rate for this protocol, finding that in the limit of large signal variance and high channel efficiency it approaches twice that of previous coherent state quantum key distribution schemes. We have considered a possible eavesdropping attack in the form of a simple feed forward scheme, which has provided us with an upper bound on the secret key rate. An important advantage of our simultaneous quadrature measurement protocol is the increase in total bandwidth. The absolute information rate, in bits/second, can be expressed as $I = W \log_2(1+S/N)$ [53], where $W$ is the limiting bandwidth associated with the state preparation or detection. Typically, in continuous variable quantum cryptography schemes, $W$ can be attributed to the switching time for the optical phase of the local oscillator. The simultaneous quadrature measurement scheme does not require switching, so that orders of magnitude increases in absolute secret key rates should be achievable.

In conclusion, we have shown that there is no need to randomly switch bases to achieve secure quantum key distribution. By performing simultaneous quadrature measurements in a coherent state quantum cryptography protocol, we are able to achieve a significantly larger secret key rate than that obtained by the usual single quadrature measurements. This new quantum key distribution protocol will allow simpler and higher bandwidth quantum cryptographic experiments and technological applications.

Chapter 6

# Conclusions and Future Work

> The time-travelling is just too dangerous. Better that I devote myself to study the other great mystery of the universe: women!
>
> - *Dr Emmett Brown, Back to the Future*

## 6.1 Summary

In conclusion, we have introduced a new coherent state continuous variable quantum cryptographic protocol, known as the simultaneous quadrature measurement protocol or the SQM protocol. All previous quantum cryptography schemes, both discrete and continuous, relied on a fundamental step in their protocols: basis switching. This step was thought important in order to maintain an extra amount of security from any eavesdropper. We have shown that this is not true. By eliminating the need to randomly switch bases, we have shown that Alice and Bob can still build a secure secret key. Furthermore, by measuring both quadratures simultaneously, we showed that the information rate is doubled in the limit of high variance. Our protocol will also result in a simplification of any experimental or commercial quantum cryptographic application, along with a significant increase in bandwidth. The SQM protocol can also be applied to all previous continuous variable quantum cryptographic protocols.

## 6.2 Future Work

A number of exciting future prospects have resulted from the research that was carried out in this thesis. These include adapting the simultaneous quadrature measurement protocol to other key distillation techniques such as postselection. As well as more fundamental quantum information problems such as distinguishing between quantum states.

### 6.2.1 The SQM Protocol using Postselection

In previous chapters we discussed the concepts of reconciliation and privacy amplification procedures. In this thesis we used reverse reconciliation which is just one possible way. Another reconciliation and privacy amplification technique is postselection [36]. This is when Alice and Bob post select a subset of their raw data - a subset that they know is completely secure from Eve. Postselection results in much lower information rates than reverse reconciliation, but it is able to tolerate significantly higher amounts of channel noise. We will briefly describe how postselection currently works, i.e. with Bob randomly switching between his measurement bases.

Alice prepares an ensemble of quantum states from a possible choice of four coherent states $|\alpha e^{i\theta}\rangle$ $|-\alpha e^{-i\theta}\rangle$ $|i\alpha e^{i\theta}\rangle$ or $|-i\alpha e^{-i\theta}\rangle$ which have been randomly displaced from a vacuum state centered at the origin. Alice and Bob have previously agreed upon the bit allocation where all the positively displaced states are a 0 and negative ones a 1, i.e. $|\alpha \mathrm{e}^{\mathrm{i}\theta}\rangle \rightarrow 0, |-\alpha \mathrm{e}^{-\mathrm{i}\theta}\rangle \rightarrow 1, |\mathrm{i}\alpha \mathrm{e}^{\mathrm{i}\theta}\rangle \rightarrow 0, |-\mathrm{i}\alpha \mathrm{e}^{-\mathrm{i}\theta}\rangle \rightarrow 1$. She then sends this ensemble to Bob who measures them using homodyne detection. He then randomly switches the basis used to measure the quantum states. These measurements form two gaussian distributions for both the amplitude and phase quadratures. These two distributions overlap at some point creating a region of uncertainty. The quantum communication is now over and the classical communication and thus reconciliation begins. The next step for Alice and Bob is the securing of their raw key by putting a postselected threshold on their two gaussian distributions. So they will keep any data over this threshold and allocate a bit value to it, while all data within this region will be discarded. Another important point is that because Bob and Eve's information is uncorrelated, Eve has no way of knowing what this threshold value is. Therefore the eavesdropper's information is exponentially reduced to zero.

A number of continuous variable quantum cryptographic papers have incorporated postselection into their protocols [36, 54]. However all of these rely on basis switching. Therefore future work could consist of adapting postsection to our simultaneous quadrature measurement protocol.

### 6.2.2 Other Quantum Cryptographic Proposals

As we have seen from the last chapter, the question of an optimal bound is still unresolved. An optimal bound is the maximum amount of information that Eve could obtain from measuring both quadratures simultaneously. The determination of an optimal threshold is directly linked to the concept of entropy and the accessible information of quantum states [8]. Therefore our analysis in this thesis could also be transformed using entropic relations - instead of the beam splitter equations - as a possible way of determining the optimal bound. Further research could also consist of a comparison between the SQM protocol developed in this thesis and the information rates derived from the discrete variable quantum cryptographic schemes.

### 6.2.3 The Accessible Information of Quantum States

> There can't be that much to say about two nonorthogonal states.
>
> \- *Jeff Kimble*

The accessible information of quantum states is not only applicable to quantum cryptography but to quantum information in general. The only known case for the accessible information (or mutual information)[1] is for the simple two pure state case [56]. It is not currently known for the general or mixed state cases. One of the main problems arises from the difficulty in determining what the optimal measurement basis is for N states.

[1] see [8, 55] for an introduction to accessible information.

## 6.3 Conclusion

Quantum information is one of the most exciting and rapidly developing fields in physics today. It is based on the theory of quantum mechanics and has such diverse applications as quantum computation, teleportation and quantum cryptography. This thesis has presented a new quantum cryptographic protocol that has eliminated a fundamental step that all previous protocols were dependent on. And it is an example of the many possibilities that lie ahead in quantum information theory.

# Appendix A

# Matlab Code for Feed Forward Optimization

The following code was written by Warwick Bowen. It is used in Chapter 5 to optimize the feed forward scheme.

```
% Matlab code for Optimisation of Feed Forward
%% Use for Appendix of Thesis

clear all

%Variables

num=400;    %resolution

etamin=0.0001; etamax=0.9999;
etavec=etamin:(etamax-etamin)/num:etamax;
eta=ones(size(etavec))'*etavec;     %line efficiency

epsimin=0.0001; epsimax=0.9999;
epsivec=epsimin:(epsimax-epsimin)/num:epsimax;
epsi=(ones(size(epsivec))'*epsivec)';   %Eves beam splitter

\end{alltt}




Vn=1;                   %Channel noise
Vin=99;             %Signal variance (Alice total variance = Vin+1)

%Working out what epsi's are valid

F1=(1-eta)*Vn; F2=(sqrt(1-epsi) -
(sqrt(eta)-sqrt(epsi)).*sqrt(epsi./(1-epsi))).^2 +
(sqrt(eta)-sqrt(epsi)).^2./(1-epsi);
```

```
Error=0.000001;

check=(sign(F1-F2*(1-Error))+1)/2;

%Conditional variances

VBEa=0.5*(eta*(Vin+1)+(sqrt(1-epsi)-(sqrt(eta)-sqrt(epsi)).*sqrt(epsi./(1-epsi))).^2
+ (sqrt(eta)-sqrt(epsi)).^2./(1-epsi))+0.5;
VBEb=sqrt(eta.*(1-epsi))*(Vin+1) -
sqrt(epsi).*(sqrt(1-epsi)-(sqrt(eta)-sqrt(epsi)).*sqrt(epsi./(1-epsi)))+(sqrt(eta)-sqrt(epsi))./sq
VBEc=2*((1-epsi)*Vin+2); VBE=VBEa-VBEb.^2./VBEc;

VBE=VBE+(1-check)*2^40;

VBA=0.5*(1+(1-eta)*Vn+eta);

exp=min(log2(VBE./VBA));
```

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