

# Optical Frequency Comb Generation

Génération d'un peigne de fréquence optique

Stage de recherche, MIP deuxième année

**Adrien Mahé**

**Responsable : Thomas Symul & Ping Koy Lam**

thomas.symul@anu.edu.au, +61 (0)2 6125 2810

Ping.Lam@anu.edu.au, +61 (0)2 6125 8378

Quantum Optics Group, Department of Physics, Australian National University  
Canberra, Australia

**14th January 2003–29th July 2003**

# Contents

<b>1</b>	<b>Principles of the Frequency Comb generation</b>	<b>2</b>
1.1	How to make a frequency comb . . . . .	2
1.1.1	Modulation of a laser beam . . . . .	2
1.1.2	Optical cavity . . . . .	3
1.1.3	Non-linear effects . . . . .	4
1.2	Electro-optic modulation . . . . .	4
1.2.1	Pockels Effect . . . . .	4
1.2.2	Modulation of light . . . . .	6
1.3	Optical cavity . . . . .	7
1.4	Non-linear effects . . . . .	9
1.4.1	Non-linear media and light . . . . .	9
1.4.2	Non-linear wave equation . . . . .	9
1.4.3	Second harmonic generation . . . . .	10
1.4.4	Three wave mixing . . . . .	10
1.4.5	Thermal effect . . . . .	10
1.5	Limitation to the Optical Frequency Comb . . . . .	10
<b>2</b>	<b>Rectangular cavity</b>	<b>12</b>
2.1	Design of the cavity . . . . .	12
2.1.1	Coordinates and stuff... . . . .	12
2.1.2	Reduced wave equation . . . . .	13
2.1.3	TE waves . . . . .	13
2.1.4	Partially loaded waveguide . . . . .	14
2.1.5	Finish of the cavity . . . . .	15
2.2	Preparation of the oven . . . . .	16
2.2.1	Heater and sensor . . . . .	16
2.2.2	Temperature control . . . . .	17
2.3	Test of the cavity . . . . .	17
2.3.1	Circulator . . . . .	17
2.3.2	Optical test . . . . .	18
2.4	Why doesn't it work? . . . . .	18
<b>3</b>	<b>Reentrant cavity</b>	<b>19</b>
3.1	Principle of a reentrant cavity . . . . .	19
3.1.1	Presentation . . . . .	19
3.1.2	Reduction factor . . . . .	20
3.2	Computer simulations using Superfish . . . . .	21
3.2.1	Superfish . . . . .	21
3.2.2	Equivalent crystal . . . . .	21
3.2.3	Problems solved . . . . .	22
3.2.4	Design of our cavity . . . . .	22

3.3	Oven . . . . .	24
3.4	Results . . . . .	25
3.4.1	Resonant frequencies . . . . .	25
3.4.2	Modulation . . . . .	25
3.4.3	Conclusion . . . . .	25
<b>4</b>	<b>Optics</b>	<b>27</b>
4.1	Design of the experimental setup . . . . .	27
4.1.1	ABCD Matrices . . . . .	27
4.1.2	Mirrors for the optical cavity . . . . .	27
4.1.3	Lenses: mode matching . . . . .	28
4.2	Setting up the laser . . . . .	29
4.2.1	Quarter and half wave plates . . . . .	29
4.2.2	Faraday isolator . . . . .	29
4.2.3	Beam stirrer . . . . .	29
4.3	Alignment . . . . .	29
4.3.1	Mirrors, lenses . . . . .	30
4.3.2	Optic fibers . . . . .	30
4.3.3	Cavity . . . . .	30
4.4	Cavity locking . . . . .	32
4.4.1	Principle . . . . .	32
4.4.2	Experimental realization . . . . .	32
4.5	No results . . . . .	33
<b>A</b>	<b>Mathematica programs</b>	<b>36</b>
A.1	Rectangular cavity . . . . .	36
A.2	Optical length . . . . .	36
A.3	Mirrors . . . . .	36
<b>B</b>	<b>Superfish programs</b>	<b>38</b>
B.1	Yacin's empty cavity . . . . .	38
B.2	Design of our cavity . . . . .	38
	Références . . . . .	39

## Résumé

Ce rapport présente des aspects théoriques et expérimentaux à propos de la génération d'un peigne de fréquence optique. La partie théorique présente aussi des effets non linéaires permettant une génération active de ce peigne. La partie expérimentale explique la conception et le test de deux types de modulateur électro-optique à fréquence microonde. Je donne des raisons pour l'absence de modulation avec le modulateur rectangulaire. Je donne aussi les résultats pour le modulateur réentrant, ainsi que des conseils pour en construire un plus efficace. A la fin, j'explique le montage optique et le locking de la cavité par la méthode de Pound, Drever et Hall.

## **Abstract**

This report presents theoretical and experimental aspects of optical frequency comb generation. The theoretical part also presents some non linear effects allowing active comb generation. The experimental part deals with the design and testing of two types of microwave electro optic modulator. It gives reasons about the failure of the rectangular modulator and results for the reentrant one, as well as hints to build a better one. Eventually, I explain the optical setup and the locking of the optical cavity by the Pound Drever Hall method.

**Introduction** I have spent six months in the Quantum Optic Group of the Australian National University (ANU), in Canberra. This group involves one researcher (Ping Koy Lam), one post doc (Thomas Symul) and a few PhD students (Warwick Bowen, Andrew Lance, Kirk Mac Kenzie and Magnus Hsu). It is part of a bigger group, which is the centre of excellence in Quantum Atom Optics.

The project I have undertaken there was "dynamic" frequency comb generation: starting with a laser beam of given frequency, the aim is to create as many equally spaced sidebands as possible in the frequency spectrum and to be able to choose their sizes to a certain extent. Many frequency combs have been made in the past, the idea of this project is to understand how they work and to help people who will need to use one in the future in the ANU. It should be also one of the first attempts to control the sizes of the sidebands.

Frequency combs can be used in metrology, telecommunications, spectroscopy and for short pulse lasers. Choosing the size of the sidebands can be useful to modify the shape of the pulses.

In order to build such a comb, I had to try and design a microwave cavity, since commercially available cavities were not suitable for our experiment, and it turned out to be much more difficult than I had thought in the beginning. Designing the microwave cavity involved a little bit of theory and computer programming. Testing it allowed me to deal with optics, and learn a few things on how to use a laser. Then I built and locked an optical cavity around the microwave cavity, which involves a little bit of electronics.

In a first part, we'll see how it is possible to make a frequency comb, and how we tried to make ours. The second part will deal with the first microwave cavity we built. The third part will be focused on the reentrant cavity, the second type of cavity we used to try and modulate light. The last part will deal with all the optics which were involved in this experiment, especially the design and construction of the optical cavity.

# Principles of the Frequency Comb generation

## 1.1 How to make a frequency comb

In this section, we will explain qualitatively how a frequency comb can be generated.

### 1.1.1 Modulation of a laser beam

#### Phase modulation

We want to use a laser beam, which has a given frequency, and add frequencies to it. A way to do it is to use a phase modulator, which is an electro-optic device (see section 1.2). Such a device allows to add a time-dependent phase to the beam. For a beam of frequency  $\omega$  and amplitude  $E_0$  propagating in the z direction we can write the complex electric field in the form:  $\vec{E} = \vec{E}_0 e^{i(kz - \omega t)}$ , a phase modulated beam would be for example  $\vec{E}_{mod} = \vec{E}_0 e^{i(kz - \omega t + \eta \cos(\Omega_m t + \phi))}$ , where  $\eta$  is the strength and  $\Omega_m$  the frequency of the modulation.  $\phi$  represents the phase of the electro-optic modulator compared to the laser beam.

#### Sidebands

When  $\eta$  is small enough ( $\eta \ll 1$ ), the complex electric field can be written:

$$\vec{E}_{mod} = \vec{E}_0 * (e^{i(\omega t + \eta \cos(\Omega_m t + \phi))}) \quad (1.1)$$

$$\simeq \vec{E}_0 e^{i\omega t} (1 + i\eta \cos(\Omega_m t + \phi)) \quad (1.2)$$

$$\simeq \vec{E}_0 (e^{i\omega t} + i\frac{\eta}{2} e^{i(\omega + \Omega_m)t + i\phi} + i\frac{\eta}{2} e^{i(\omega - \Omega_m)t - i\phi}) \quad (1.3)$$

As we can see in equation 1.3, a frequency spectrum analysis of the beam would show the initial frequency of the beam (the carrier), and two other frequencies of small amplitude, at  $\omega + \Omega_m$  and  $\omega - \Omega_m$ , which are called the sidebands. We can represent this spectrum on a 3-dimensional diagram, on which the carrier doesn't move and the sidebands rotate around the frequency axis (see fig 1.1), the rotation representing the phase of the sidebands compared to the carrier. (The sidebands thus rotate around the frequency axis in opposite directions and at a frequency  $\Omega_m$ )

#### Bessel functions

More exactly, we can expand the phase modulated field thanks to Bessel functions [1],

$$\vec{E}_{mod} = \vec{E}_0 e^{i\omega t} e^{i\eta \cos(\Omega_m t + \phi)} \quad (1.4)$$

$$= \vec{E}_0 e^{i\omega t} \sum_{n=-\infty}^{\infty} i^n J_n(\eta) e^{in(\Omega_m t + \phi)} \quad (1.5)$$

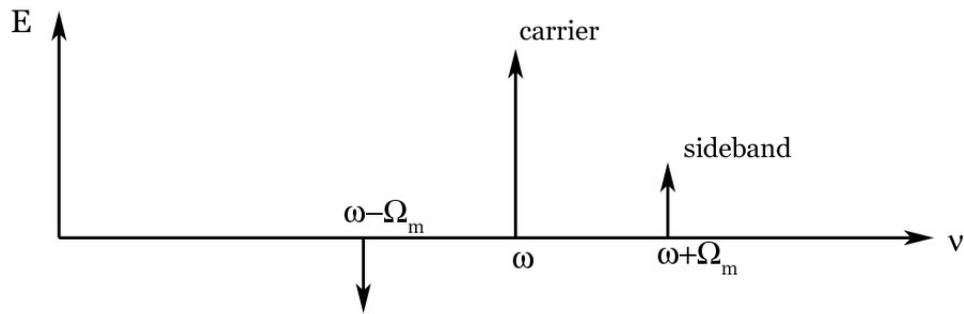


Figure 1.1: Sidebands representation

where  $J_n$  is the Bessel function of the first kind. We have  $J_{-m}(\eta) = (-1)^m J_m(\eta)$ , and these results are coherent with equations 1.3. The first six Bessel functions are plotted in fig 1.2

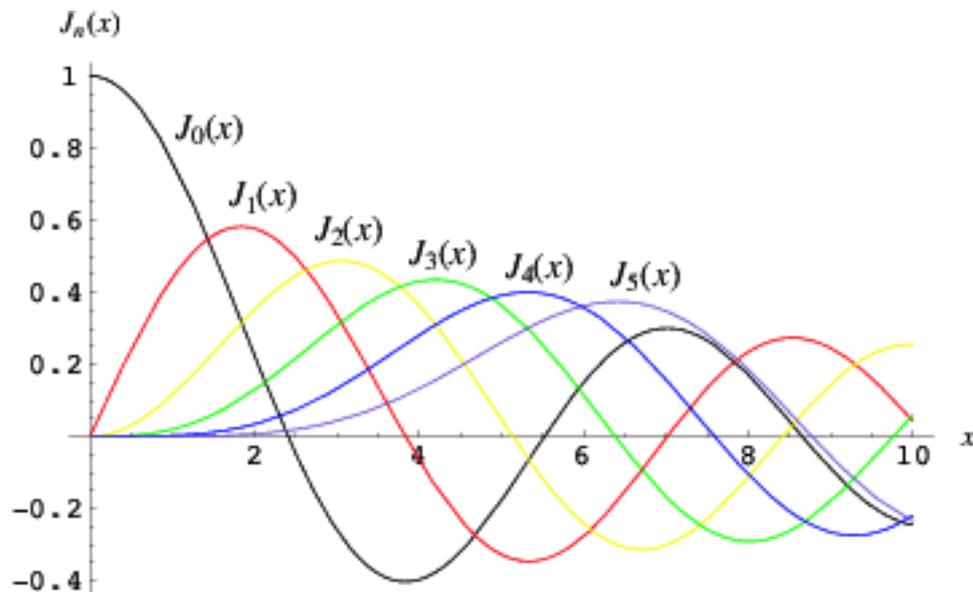


Figure 1.2: Bessel functions

We can see that phase modulation is equivalent to adding frequencies in the beam's spectrum. The higher the efficiency of the modulation, the more frequencies can be measured. Nevertheless, high efficiencies are hard to achieve, and to get as many frequencies as possible, it is often necessary to have multiple passes in the electro-optic modulator.

### 1.1.2 Optical cavity

In order to achieve high modulation, we can put a mirror on each side of the electro-optic modulator (see fig 1.3). We obtain thus a Fabry-Perot cavity. We can choose the reflectivity of the mirrors so that we don't damage the electro-optic modulator and that have enough passes to have a broad enough comb. We also need to choose their radii so that the cavity is stable. We need to be careful though to choose the cavity so that it works well with the electro-optic modulator (see section 1.3)

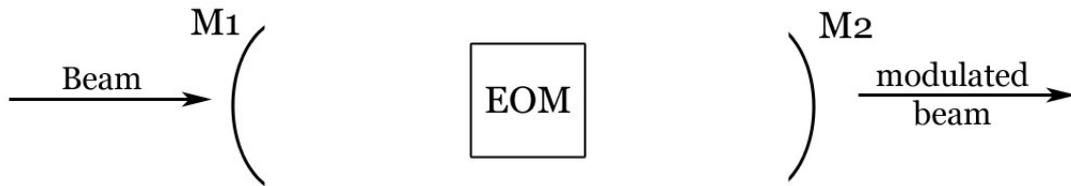


Figure 1.3: electro-optic modulator inside a Fabry-Perot

### 1.1.3 Non-linear effects

Here is the originality of our work. We want to be able to choose to a certain extent the shape of the comb. In order to achieve that, we can use the properties of several crystals, which are able to convert two photons in a single photon (second harmonic generation, or SHG) which has the sum of their energies and then convert this single photon back into two individual photons whose energies add to the single photon energy. The interesting point is that the second process is temperature dependent. This means that at a given temperature, the high energy photon will rather give two photons with equal energies and at another temperature, it will most probably give two photons with slightly different energies, the conservation of energy being respected.

## 1.2 Electro-optic modulation

The electro-optic effect is the change of a material's refractive index resulting from the application of a DC or low frequency<sup>1</sup> electric field [2]. The property of such materials can be used to modulate light.

### 1.2.1 Pockels Effect

#### in an isotropic medium

The refractive index of an electro-optic medium is a function of the applied electric field. Since this effect is usually small, we can expand it near  $\vec{E} = \vec{0}$ :

$$n(E) = n(0) + a_1 E + 1/2 a_2 E^2 + \dots \quad (1.6)$$

The coefficients  $a_1$  and  $a_2$  are not conventionally used. When  $a_1$  is non zero (which is the case for our Mg doped  $LiNbO_3$  crystal), further terms are negligible.  $a_1$ 's value usually lies in the range 1 to 100 pm/V : 10kV applied to a crystal 1cm thick would result in a variation of  $n$  of about  $10^{-5}$ .

#### in an anisotropic medium

**Electric permittivity and permeability** An anisotropic medium is described by the electric permittivity tensor  $\hat{\epsilon}$ , which links the electric flux density  $\vec{D}$  and the electric field of light:

$$\vec{D} = \hat{\epsilon} \vec{E}_{light} \quad (1.7)$$

<sup>1</sup>this excludes considering the interaction of light with itself or with another beam

$\hat{\epsilon}$  is a rank 2 tensor, and is proportional to the unity matrix in the case of an isotropic medium. It is always possible to find a coordinate system for which  $\hat{\epsilon}$  is diagonal. Thanks to the symmetries of the crystals i have used, we can find two axis with "ordinary" index  $n_o$  and one axis, called the optic axis, with an "extraordinary" index  $n_e$  (indexes and electric permittivity are linked by the following formula:  $n_i = \sqrt{\frac{\epsilon_i}{\epsilon_0}}$ ). They are called uniaxial crystals.

Since we can make  $\hat{\epsilon}$  diagonal and its principal values are non zero, we can define  $\hat{\eta}$  so that

$$\epsilon_0 \overrightarrow{E}_{light} = \hat{\eta} \overrightarrow{D} \quad (1.8)$$

When a steady electric field is applied to a crystal, elements of  $\hat{\eta}$  are altered, so that each of the nine elements  $\eta_{ij}$  becomes a function of each component of  $\overrightarrow{E}$ . We can expand each element in a Taylor's series about  $\overrightarrow{E} = \overrightarrow{0}$ :

$$\eta_{ij}(\overrightarrow{E}) = \eta_{ij}(\overrightarrow{0}) + \sum_{k=1}^3 r_{ijk} E_k + \sum_{k,l} \zeta_{ijkl} E_k E_l + \dots \quad i, j, k, l = 1, 2, 3 \quad (1.9)$$

where  $r_{ijk} = \frac{\partial \eta_{ij}}{\partial E_k} |_{E=0}$  and  $\zeta_{ijkl} = \frac{1}{2} \frac{\partial^2 \eta_{ij}}{\partial E_k \partial E_l} |_{E=0}$ .  $\eta_{ij}$  are known as the linear electro-optic or Pockels coefficients. We can then use properties of symmetry of  $\hat{\eta}$  and symmetries of the crystal to see that only 5 of these coefficients are independent for a *LiNbO<sub>3</sub>* crystal [2].

**Index ellipsoid** For anisotropic media, an equation called the index ellipsoid is commonly used: it can be shown that a surface defined by

$$\sum_{ij} \eta_{ij} x_i x_j = 1 \quad i, j = 1, 2, 3 \quad (1.10)$$

is invariant to the choice of coordinate system, and carries all information about the  $\hat{\eta}$  tensor (which has six degrees of freedom: the directions of the principal axis and the values of the principal refractive index). Using the principal axes as a coordinate system, the index ellipsoid is described by

$$\frac{x_1^2}{n_1^2} + \frac{x_2^2}{n_2^2} + \frac{x_3^2}{n_3^2} = 1 \quad (1.11)$$

This surface can be used to determine the index seen by light travelling in any direction and with any polarization. We have only used light travelling along one of the principal axis with ordinary index. Thus (see [2],[3] for a more general view) light can be decomposed in two beams, one with polarization along the other ordinary index axis which will have  $n_o$  as index and one with polarization along the extraordinary axis which will travel at  $c/n_e$ .

**Index change in *LiNbO<sub>3</sub>*** Using 1.9 in 1.10 and tables for Pockels coefficient available in literature (for example [2], or [4]) we can show that for a low frequency or DC electric field along the optic axis the index ellipsoid becomes

$$\left(\frac{1}{n_o^2} + r_{13}E\right)(x_1^2 + x_2^2) + \left(\frac{1}{n_e^2} + r_{33}E\right)x_3^2 = 1 \quad (1.12)$$

Thus, the ordinary and extraordinary indices are now given by

$$\frac{1}{n_o^2(E)} = \frac{1}{n_o^2} + r_{13}E \quad (1.13)$$

$$\frac{1}{n_e^2(E)} = \frac{1}{n_e^2} + r_{33}E \quad (1.14)$$

Since usually  $r_{13}E$  and  $r_{33}E$  are small compared to  $1/n_e^2$  and  $1/n_o^2$  we can make the following approximations:

$$n_o(E) \simeq n_o - 1/2n_o^3r_{13}E \quad (1.15)$$

$$n_e(E) \simeq n_e - 1/2n_e^3r_{13}E \quad (1.16)$$

We saw in this part that an electric field applied along the optic axis of the uniaxial  $LiNbO_3$  crystal we used didn't change the crystal principal axes but its refractive indices are modified in a way very similar to the way indices are modified for a an isotropic medium. For our crystal the effect on the extraordinary axis is more important than the effect on the ordinary axis.

## 1.2.2 Modulation of light

As we saw in 1.2.1, the Pockels effect seems really small, but a variation of  $10^{-5}$  in the index can entail provide a phase shift of a few radians if applied over a length of a few  $10^5$  wavelengths.

### Phase shift with a DC voltage

An electric field can be applied to the crystal by submitting it to a voltage  $V$  across his height  $d$ . A wave travelling the length  $L$  of the crystal undergoes a phase shift that is usually written:

$$\varphi = \varphi_0 - \pi \frac{V}{V_\pi} \quad (1.17)$$

where  $\varphi_0 = 2\pi nL/\lambda$ ,  $\lambda$  is the wavelength of the light and

$$V_\pi = \frac{d}{L} \frac{\lambda}{rn^3} \quad (1.18)$$

is the voltage needed to generate a phase shift of  $\pi$  called the halfwave voltage. We can then calculate this halfwave voltage for a  $LiNbO_3$  crystal thanks to tables given in litterature <sup>2</sup>. If we choose to have a linearly polarized optical wave travelling along the optic axis (longitudinal modulator)  $V_\pi \simeq 5 \cdot 10^3 V$ , whereas if the wave travels along an ordinary axis and is polarized along the optic axis (transverse modulator), then the halfwave voltage is only of around  $200 V$ . This is why we used the crystal as a transverse modulator.

### Phase modulation with a low frequency voltage

We will now assume that if the frequency is not too high, we can still use the formulas 1.15 and 1.16 for the indices, except that they become variable with time. We will also assume that there is an electromagnetic wave travelling at speed  $v_\mu$  inside the crystal (proportionnal for example at  $\frac{V}{2} \sin(\Omega_m(t - z/v_\mu))$ ), creating a travelling wave of modified indices in the crystal. Inside the crystal, light with polarization along the optic axis travels at speed  $v_{light} = c/n_e$ , where we can consider  $n_e$  to be constant since its variation causes second order phase modulation which can be neglected. If we consider a small part of an optical ray, travelling at this speed  $v_{light}$ , say in the

---

<sup>2</sup>The values for the Pockels coefficient vary slightly depending on the source so I took average values

same direction as the low frequency voltage, it will see when going through the crystal an index varying as  $\sin(\Omega_m(t' + z(1/v_{light} - 1/v_\mu)))$ , where  $t'$  is the time when this "part of optical ray" enters the crystal. Thus the optical length  $\delta$  this ray will go through is (assuming the crystal starts at  $z = 0$  and has a length  $L$ ):

$$\delta = \int_0^L [n_e - \frac{1}{2}n_e^3r_{13}\frac{V}{2d}\sin(\Omega_m(t' + z(1/v_{light} - 1/v_\mu)))]dz \quad (1.19)$$

The first term in this equation is not time dependent and is the phase the beam acquires when going through the crystal without any low frequency voltage applied. The second is the one which will give modulation and it can be shown from this formula that:

$$\delta = n_eL - \frac{1}{2}\frac{n_e^3r_{13}V/d}{\Omega_m(1/v_{light} - 1/v_\mu)}\sin(\Omega_m(t' + \frac{L}{2}(\frac{1}{v_{light}} - \frac{1}{v_\mu})))\sin(\Omega_m\frac{L}{2}(\frac{1}{v_{light}} - \frac{1}{v_\mu})) \quad (1.20)$$

Since the optical length and the phase added to the light are linked by  $\phi = 2\pi\frac{\delta}{\lambda}$ , we can identify the first sine function in equation 1.20 to the phase modulation we wanted to achieve and which has been described in the first section of this chapter. We can thus rewrite the phase modulation for a single pass in a phase modulator which has a travelling electric wave as

$$\varphi = \eta\sin(\Omega_mt' + \phi_0) \quad (1.21)$$

where

$$\phi_0 = \Omega_m\frac{L}{2}(\frac{1}{v_{light}} - \frac{1}{v_\mu}) \quad (1.22)$$

$$\eta = \frac{\pi\beta n_e^3r_{33}EL}{\lambda} \quad (1.23)$$

$$\beta = \frac{1}{2}\frac{\sin(u)}{u} \quad (1.24)$$

$$u = \frac{\Omega_mL}{2}(\frac{1}{v_{light}} - \frac{1}{v_\mu}) \quad (1.25)$$

In these equations,  $\phi_0$  is just a constant phase which doesn't really matter,  $\eta$  is the depth of modulation introduced in a previous section,  $\beta$  is called the reduction factor and shows that the difference between the speed of light and the velocity of the electromagnetic wave applied to the crystal is important, since for certain values of this difference, no modulation is possible, as can be seen on fig 1.4.

The figure shows that it is important to match as well as possible the speed of the light and the velocity of the low frequency electromagnetic wave (which we will from now on call the microwave wave since that is what it was in the experiment) since the modulation depth is best when these two speeds are equal.

The influence of the crystal length will be discussed later in section 3.1.2.

### 1.3 Optical cavity

Two conditions need to be satisfied so that the optical cavity help broadening the spectrum:

- first, the carrier and the sidebands must be resonant frequencies of the Fabry-Perot cavity: if the free spectral range (FSR) of the fabry-Perot cavity is  $\Delta f$ , then we must build the cavity so that [7]

$$n\Delta f = \frac{\Omega_m}{2\pi} \quad (1.26)$$

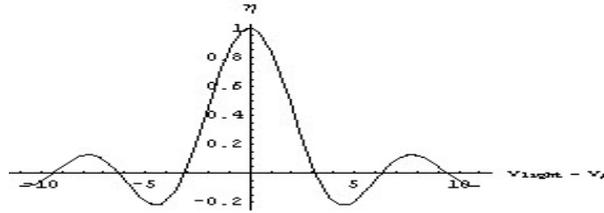


Figure 1.4: Strength of the modulation  $\eta$  (arbitrary units) plotted against the difference of velocity of the light and the low frequency electromagnetic wave

where  $n$  is an integer.

- then, we must ensure that the modulation adds at each pass in the electro-optic modulator. for example, we can imagine a cavity for which the phase would be modulated by  $\eta \cos(\Omega_m t + \phi_1)$  at the first pass and then  $\eta \cos(\Omega_m t + \phi_2)$  at the second pass and so on with different  $\phi_i$  at each pass. We must thus make sure that the beam always comes in the modulator in phase with the modulation. If we assume the electro-optic modulator has a standing wave inside (which was the case in the one i built), we can decompose this standing wave into one propagating and one contrapropagative waves, the one propagating in the same direction as the beam being the only one important thanks to eq 1.24. We must then ensure that the difference between the time  $t'_1$  when the beam enters the crystal for the first time and the times  $t'_i$  when the beam enters the crystal for the  $i^{th}$  time are such that

$$\Omega_m(t'_i - t'_1) = 2\pi * n' \tag{1.27}$$

where  $n'$  is an integer, so that the modulation depth adds at each pass (see eq 1.21). If we assume that the electro-optic modulator is standing in the middle of the optical cavity, then  $t'_i - t'_1 = i * L_{opt}/c = \frac{i}{2\Delta f}$ . So the condition comes down to

$$\frac{\Omega_m i}{2\pi} = 2n' \Delta f \tag{1.28}$$

which is always fulfilled when eq 1.26 is satisfied.

The second condition may not be always realized, since it is very hard in practice to put the electro-optic modulator exactly in the middle of the optical cavity. We have to hope that the modulation of light "on the way back" is not the exact opposite of the modulation it got when first going through it.

As for the first condition, it is hard to understand what actually happens inside the optical cavity, since sidebands only appear in the beam once it is inside the optical cavity. My guess is that if sidebands do not fit exactly in the Free Spectral Range pattern of the optical cavity, then the modulation they get at each pass has a random phase, thus preventing them from broadening the comb.

## 1.4 Non-linear effects

This section will be a brief introduction to some other properties our crystal has. It will be short because i didn't have enough time to use these properties experimentally, although they are part of the project. More complete informations can be found in [3] [2]

We can notice that the same formalism allows us to understand both the Pockels effect and the non-linear effects.

### 1.4.1 Non-linear media and light

The crystal used in this experiment is a non-linear medium, which generates some surprising properties. For instance, the principle of superposition can be violated and new frequencies can be generated. We must note that the crystal used has a range of temperature (including room temperature) where non linear effects cannot be seen.

As opposed to a linear medium, a non-linear medium has a non-linear relation between the polarization density  $\vec{P}$  and the electric field  $\vec{E}$  which can now have any frequency, including optical frequencies. Actually, the relation is usually linear for small electric fields, but becomes non-linear when the electric field reaches values comparable with interatomic electric fields.

Assuming that the response of the medium is instantaneous (which is equivalent to the crystal being transparent) for the frequencies considered we can write the polarization density :

$$P_i = \epsilon_0 \left( \sum_j \chi_{ij}^{(1)} E_j + \sum_{j,k} \chi_{ijk}^{(2)} E_j E_k + \dots \right) \quad (1.29)$$

We intend to use the crystal on a normal mode (polarization along the optic axis), and we can write eq 1.29

$$P = \epsilon_0 (\chi^{(1)} E + \chi^{(2)} E^2 + \dots) \quad (1.30)$$

### 1.4.2 Non-linear wave equation

Maxwell's equations for an arbitrary homogeneous medium give us the wave equation:

$$\nabla^2 E - \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} = \mu_0 \frac{\partial^2 P}{\partial t^2} \quad (1.31)$$

If we write  $P$  as the sum of the linear term  $\epsilon_0 \chi E$  and a non-linear term  $P_{NL} = \epsilon_0 (\chi^{(2)} E_0^2 + \dots)$ , the wave equation becomes:

$$\nabla^2 E - \frac{n^2}{c^2} \frac{\partial^2 E}{\partial t^2} = \mu_0 \frac{\partial^2 P_{NL}}{\partial t^2} \quad (1.32)$$

where  $n^2 = 1 + \chi$ .

We will solve this equation using the Born approximation: we suppose that an optical wave arrives in the non-linear media, creates a non-linear polarization density which radiates an optical field and so on. We will only make the first step in the next sections, more quantitative dealing calculation can be found in [2]

### 1.4.3 Second harmonic generation

We consider the term  $P_{NL}$  with only the order two non-linearity, which is to say

$$P_{NL} = \epsilon_0 \chi^{(2)} E^2 \quad (1.33)$$

For an electric field  $E(t) = \Re(E_0(\omega)e^{i\omega t})$ , using a trigonometric formula, eq 1.33 becomes

$$P_{NL}(t) = \epsilon_0 \frac{\chi}{2} (|E(\omega)|^2 + \Re(E^2(\omega)e^{i2\omega t})) \quad (1.34)$$

Using this formula in the wave equation 1.32, we see that an electric field at frequency  $2\omega$  is generated. We can see that the intensity of the second harmonic wave generated is proportionnal to the square of the intensity of the incident light (as long as the first Born approximation is valid, which is the case for small second harmonic generation). We can also note that there is a steady polarization density across the crystal which results in a dc voltage across the crystal.

### 1.4.4 Three wave mixing

We now consider an incident wave which has two harmonic frequencies:  $E(t) = \Re(E_{0,1}(\omega)e^{i\omega_1 t} + E_{0,2}(\omega)e^{i\omega_2 t})$  When we square this to have  $P_{NL}$ , we obtain 5 frequencies: 0,  $2\omega_1$ ,  $2\omega_2$ ,  $\omega_+ = \omega_1 + \omega_2$ , and  $\omega_- = \omega_1 - \omega_2$ . Light is generated as explained in the previous section at these frequencies, although not all of these frequencies are actually to be generated at the same time, because different conditions apply (phase matching condition for example). This phenomenon is used in many applications such as parametric optical oscillators, parametric amplifiers...

### 1.4.5 Thermal effect

The reason why it is interesting for our experiment is that when we have sidebands (frequency  $\omega + n * \Omega - m$ ) thanks to the Pockels effect, we can generate second harmonic (frequency  $2\omega$ , called green light, since the laser used is an infrared laser at 1064nm.), which then can give 2 photons at  $\omega \pm n * \omega_m$ . We can thus obtain a comb which has a shape not entirely given by Bessel functions. And we can even tune the shape of the comb since what frequencies a photon at  $2\omega$  is more likely to give is temperature dependent: this property is described by Sellmeyer's equation.

## 1.5 Limitation to the Optical Frequency Comb

With Pockels effect, it seems that provided we use very reflective mirrors for the optical cavity, we could generate an infinite comb. Previous experiments show that the span is usually of order 10 *Thz* or less, for example 7.6 *Thz* in [6], or 3 *Thz* in [5]. We can give two reasons for that:

- first, the energy of light is limited and there are losses in the crystal (around 0.1% per centimetre, which is quite a lot considering that we have used a 2 cm long crystal with 99% reflective mirrors for the optical cavity), which decreases the available power at each pass. Since we want at least one photon in each tooth of the comb, the limitation of energy available limits the span of the comb.
- but this limit is not reached, what really limits an optical frequency comb is the fact that the crystal index  $n_e$  is frequency dependant, thus the free spectral range of the cavity changes

slightly for each frequency, whereas the modulation frequency is always the same. So, when the comb reaches a given span, the free spectral range of the optical cavity and the modulation frequency of the electro-optic modulator are not matched anymore and no more teeth can be created. This condition is actually dealt with in [6]

We've seen in this chapter what an optical frequency comb was and a theoretical way to build it. The next two chapters will deal with how to make an electro-optic modulator at a frequency of a few GHz, which was the first step of my project and turned to be the longest and hardest, probably because it has a lot more to do with microwave engineering than with optics.

# Rectangular cavity

The objective I was given was to phase modulate a laser beam at a frequency of roughly  $3\text{ GHz}$ . Such frequencies cannot be easily achieved by simply putting the crystal between two electrodes connected to a microwave generator. We had to find a different way of achieving it. Our idea was to build a rectangular waveguide with the crystal inside, so that there would be a travelling microwave in the crystal. The tricky part is to choose the correct dimensions for the cavity so that the optical and microwave waves propagate at the same speed.

## 2.1 Design of the cavity

### 2.1.1 Coordinates and stuff...

The coordinates we used are shown in fig 2.1 and fig 2.2.

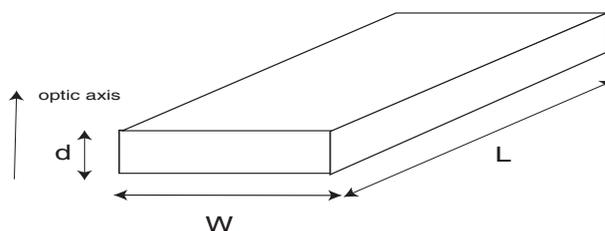


Figure 2.1: dimensions of the crystal

We were given two identical crystals which had a length  $L = 20\text{ mm}$  along the  $z$ -axis, a width  $W = 3\text{ mm}$  along the  $x$ -axis, and thickness  $d = 1.5\text{ mm}$  along the optical axis ( $y$ -axis). The

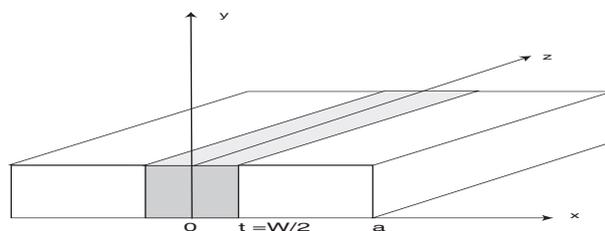


Figure 2.2: Dimensions of the microwave cavity

dimensions of the microwave cavity we want to design are  $2a$  along the  $x$  axis, and  $d$  along the  $y$ -axis: the crystal and the microwave cavity both have the same height. We will use  $t = W/2$  for the calculations in the next sections.

### 2.1.2 Reduced wave equation

Microwave engineers use waveguides to transmit microwaves with low losses. Some of them have a rectangular shape. Our idea is to couple microwaves in such a waveguide loaded with the crystal. Let us consider a medium of index  $n = \sqrt{\epsilon_r}$  for the microwave.

For an harmonic electric and magnetic field, Maxwell's equations give Helmholtz equation:

$$\Delta \vec{H} + k^2 \epsilon_r \vec{H} = 0 \quad (2.1)$$

where we do not consider any time variation of  $\vec{H}$ . Since we want the microwave to travel along the z-axis (like the laser), we will consider an electric field:

$$\vec{H}(x, y, z) = (\overrightarrow{h(x, y)} + \vec{u}_z h_z(x, y)) e^{-i\beta z} \quad (2.2)$$

where  $h_z(x, y)$  is the transverse magnetic field. We can consider a similar equation for the electric field. With such notation, we can infer the reduced wave equation from eq 2.1

$$\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k_c^2 \right) h_z = 0 \quad (2.3)$$

where  $k_c^2 = k^2 \epsilon_r - \beta^2$  is called the cutoff wavenumber (a wave with a frequency lower than the corresponding cutoff frequency cannot travel in the waveguide). This relation will prove useful to calculate the fields in a rectangular waveguide.

### 2.1.3 TE waves

Transverse electric (TE) waves are characterized by the fact that they have no electric field along the z-axis. We want to use one of these because they have an electric field along the optic axis and because they have the lowest cutoff frequency.

#### Transverse electric and magnetic fields for a TE wave in a rectangular waveguide

Using Maxwell's equations in a source free region, and assuming an  $e^{-i\beta z}$  z dependence of the electric and magnetic fields, we find the following relations for the four transverse field components as a function of the magnetic field component  $H_z$  [8]

$$H_x = \frac{-i\beta}{k_c^2} \frac{\partial H_z}{\partial x} \quad (2.4)$$

$$H_y = \frac{-i\beta}{k_c^2} \frac{\partial H_z}{\partial y} \quad (2.5)$$

$$E_x = \frac{-i\omega\mu}{k_c^2} \frac{\partial H_z}{\partial y} \quad (2.6)$$

$$E_y = \frac{i\omega\mu}{k_c^2} \frac{\partial H_z}{\partial x} \quad (2.7)$$

where  $\omega$  is the frequency of the harmonic field and  $\mu$  the relative permeability of the material.

### Longitudinal magnetic field for a TE wave in a rectangular waveguide

We can see that knowing  $h_z$  (or  $H_z$ ) allows us to find all the values of the electromagnetic wave. Using eq 2.3, and the method of separation of variables ( $h_z(x, y) = X(x)Y(y)$ ), we find:

$$\frac{d^2X}{dx^2} + k_x^2X = 0 \quad (2.8)$$

$$\frac{d^2Y}{dy^2} + k_y^2Y = 0 \quad (2.9)$$

where  $k_x^2 + k_y^2 = k_c^2$ .

Thus, the general solution for  $h_z$  is:

$$h_z(x, y) = (A\cos(k_x x) + B\sin(k_x x))(C\cos(k_y y) + D\sin(k_y y)) \quad (2.10)$$

The different constants are found thanks to boundary conditions and we can see that TE waves travelling in a rectangular waveguide with a thickness  $c$  and a width  $d$  can't take just any frequency:  $\beta = \sqrt{k^2\epsilon_r - \left(\frac{m\pi}{c}\right)^2 - \left(\frac{n\pi}{d}\right)^2}$  with  $n$  and  $m$  integers. This shows that there is a cutoff frequency for each  $TE_{m,n}$  mode.

#### 2.1.4 Partially loaded waveguide

In this section, we consider the actual crystal in a cavity as shown in fig 2.2. Since geometry is uniform in  $y$ , we will look for  $TE_{m,0}$  modes that have no  $y$  dependence. We found that there were three conditions which needed to be respected.

##### Phase matching

In order to have a travelling wave, we need to match the phase of the travelling waves outside and inside the crystal at the  $x = \pm t$  interfaces. This means that we want

$$\beta = \sqrt{\epsilon_r k^2 - k_c^2} \quad (2.11)$$

$$\text{and } \beta = \sqrt{k^2 - k_a^2} \quad (2.12)$$

where  $k_c$  and  $k_a$  are the cutoff wavenumbers respectively in the crystal and in the air. This gives us a first condition.

##### Boundary conditions

We must also ensure the continuity of the fields at the interfaces. We can use eq 2.10 in each part outside the crystal and inside the crystal.

$$h_z = \begin{cases} A\cos(k_c x) + B\sin(k_c x) & \text{for } -t \leq x \leq t \\ C\cos(k_a(a-x)) + D\sin(k_a(a-x)) & \text{for } t \leq x \leq a \\ E\cos(k_a(a+x)) + F\sin(k_a(a+x)) & \text{for } -a \leq x \leq -t \end{cases} \quad (2.13)$$

which, using eq 2.7 gives the following values for  $E_y$

$$e_y = \begin{cases} \frac{i\omega\mu}{k_c}(-A\sin(k_c x) + B\cos(k_c x)) & \text{for } -t \leq x \leq t \\ \frac{i\omega\mu}{k_a}(C\sin(k_a(a-x)) - D\cos(k_a(a-x))) & \text{for } t \leq x \leq a \\ \frac{i\omega\mu}{k_a}(E\cos(k_a(a+x)) + F\sin(k_a(a+x))) & \text{for } -a \leq x \leq -t \end{cases} \quad (2.14)$$

We can now get rid of a few constants:

- $E_y = 0$  when  $x = \pm a$ , which entails  $D=F=0$
- using the symmetry about the plane  $x=0$ , we have  $A=0$  and  $C=E$

The continuity condition at  $x = t$  becomes (the condition for  $x = -t$  is the same since we already used a symmetry argument) :

$$\begin{cases} B \cos(k_c t) = \frac{k_z}{k_a} C \sin(k_a(a-t)) & \text{for } e_y \\ B \sin(k_c t) = C \cos(k_a(a-t)) & \text{for } h_z \end{cases} \quad (2.15)$$

It is an homogeneous set of equations, so to have non zero solutions, determinant must vanish:

$$\tan(k_c t) = \frac{k_a}{k_c} \frac{1}{\tan(k_a(a-t))} \quad (2.16)$$

### Velocity phase matching

As we saw in section 1.2.2, in order to have a good modulation depth, we need to match the microwave phase velocity (which modulates the phase) with the light group velocity (which is the speed at which light propagates in the crystal). This gives us the last condition I found:

$$\frac{c}{n_e} = \frac{\omega_\mu}{\beta} \quad (2.17)$$

### Summary

We know the values for  $c$ ,  $n_e$  and  $\epsilon_r$  and the set of three equations gives us the values of  $k_c$ ,  $k_a$  and most importantly  $a$  as a function of  $\omega_\mu$ . We chose  $\omega_\mu$  so that it was close to  $3 \text{ GHz}$  and so that the microwave field could also be resonant along the  $z$ -axis. We also want to have a frequency for the microwave such that we can build an optical cavity around it with a normal size (a few to ten centimetres roughly). The calculations were made using a mathematica program given in A.1. We found the following results for a  $TE_{101}$  mode:

- the width of the cavity should be  $2a = 16,79 \text{ mm}$
- the resonant frequency should be  $3.363 \text{ GHz}$
- the length of the optical cavity can be  $64.6 \text{ mm}$  if we take  $f = 2\Delta f$  (see A.2 for mathematica program)

It turns out that the field is evanescent in the region outside the crystal.

### 2.1.5 Finish of the cavity

I sent these results to the workshop so that they could build the cavity for me. In the end, the cavity was made of 4 parts: the post, which included walls on the sides, the lid, and two caps that we could remove and which had holes ( $1 \text{ mm}^2$ ) for the laser to go through as can be seen on fig 2.3.

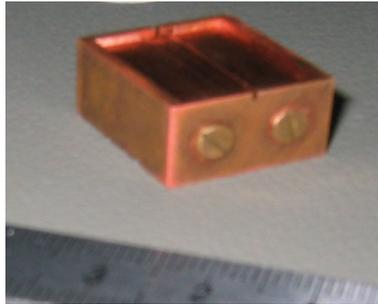


Figure 2.3: Picture of the rectangular microwave cavity

### Material used

We chose to use copper for this cavity, since it is a very reflective material for microwaves (which produces a high  $Q$  factor for the cavity) and its thermal coefficient expansion is small for a metal: one of the concerns we had is that when we heat the cavity (around  $120^{\circ}\text{C}$  if we are to produce green light) it can expand, stress and even crack the crystal.

But it is not a good idea to use such a material without coating it in some way because our cavity soon became rusty.

### Fitting the crystal in the cavity

As can be seen in fig 2.3, the height of the cavity proved to be a little bit too small for the crystal, so we asked the workshop to dig in both the post and lid a little bit to allow the crystal to fit in. Moreover, it makes the crystal more stable in the cavity: it doesn't move when someone bangs the door. The geometry is a little bit changed, but i hoped it wouldn't change too much the resonant frequency.

### antennas

We asked the workshop to drill holes in the lid so that we could fit a SMA connector in the cavity. I welded a piece of wire to this connector so that we had an antenna in the cavity. It is rather difficult to prevent the antenna from touching the lid, which has to be avoided since it creates a shortcut. We put the antenna as close as possible to the crystal and at the middle of the length of the cavity, where the electric field is bigger.

## 2.2 Preparation of the oven

For the reasons explained in the first chapter, we wanted to be able to temperature control the crystal. For this rectangular cavity, i used one of the ovens usually used in the lab. Its size was just the size of the rectangular cavity we built. A picture of it can be seen on fig 2.4

### 2.2.1 Heater and sensor

The heating was made thanks to two parallel sets of two  $1\ \Omega$  and one  $0.82\ \Omega$  resistors in series. These resistors are designed to be able to give  $6\ \text{W}$  at  $70^{\circ}\text{C}$ . The temperature controller we used (Wavelength Electronics LFI-3751) was designed to be able to deliver  $40\ \text{W}$  with a maximum output current of  $5\ \text{A}$ . The reason why I used two  $0.82\ \Omega$  resistors is that many resistors were a bit

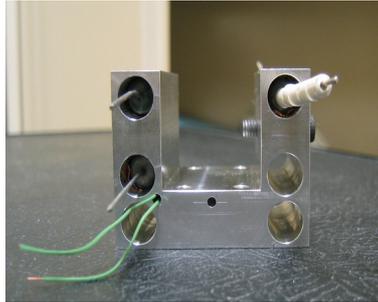


Figure 2.4: Picture of the oven used for the rectangular cavity

too big to fit inside the holes (see fig 2.4 and created short circuits. I then put some temperature and electric resistant silicone rubber (Silastic RTV Silicone rubber 744).

The sensor used was a thermistor which is a resistance that varies with temperature. As an example, the one i used for this oven was  $470\text{ k}\Omega$  at  $25^\circ\text{C}$  and  $10\text{ k}\Omega$  at  $100^\circ\text{C}$ . We placed the thermistor as close as possible from the resistors (see fig 2.4) so that the time response could be as small as possible.

### 2.2.2 Temperature control

To have this temperature controller work we only need to supply 3 values of the thermistor for 3 different temperatures. We then set a maximum output current to prevent it from overloading the resistors when it reaches the set temperature (to stick to a given temperature, the current is smaller).

For this particular setup, one problem is that the temperature controller cannot measure resistance above  $500\text{ k}\Omega$ , so I had to 'preheat' the oven by putting my hands around it for a little while before i could turn the temperature controller on.

## 2.3 Test of the cavity

We used a microwave signal generator (Rhode & Schwartz SMR 20) and a microwave amplifier to feed the cavity. We used two different methods to look for a resonant frequency of the cavity.

### 2.3.1 Circulator

The first idea we had to test the cavity was to use a circulator, which is a device with three connectors. In the first one comes the signal, which is directed to the second connector (which is connected to the cavity) and the third connector gives the reflected power from the second connection (when there is impedance mismatches, most of the power is reflected.) The assumption was that when the cavity would resonate, it would leak more microwave, thus reflecting less of the power. We sent the reflected power to a spectrum analyzer (Hewlett Packard E4405B) and using the 'hold max' button, i scanned the cavity from 2 to 4  $\text{GHz}$  which is the range in which the circulator can work. It is important to compare the data thus acquired with data acquired with the second connector unconnected because there appears to be many 'resonances' which are probably due to the apparition of standing waves in the cables since they exist in both cases, with or without the cavity connected.

### 2.3.2 Optical test

I mode matched the beam inside the cavity so that the waist was about  $0.82 \mu\text{m}$  in the center of the crystal. We had a fast response photodiode (able to see modulation up to  $10 \text{ GHz}$ ). So I had to transform the phase modulation in amplitude modulation, thanks to a quarter wave plate before the cavity (which makes the polarization circular) and a half wave plate after the cavity with a polarizing beam splitter (see 4.2.1). The signal from the photodiode was sent to the spectrum analyzer and i scanned the frequency from 2 to  $4 \text{ GHz}$  very carefully ( $1 \text{ MHz}$ ) since the maximum Q factor i estimated was 2000. Then i scanned more roughly the cavity from 1 to  $6 \text{ GHz}$ .

To make sure the microwave generator and the photodiode were working, i used a New Focus Reentrant cavity at  $4 \text{ GHz}$ , aligned it and checked for modulation, which i could see, but was very hard to find.

It is a very long process, since the modulation expected is weak, and there is a lot of pick up: microwaves go straight from the generator to the spectrum analyzer.

After 2 months spent on this cavity, i stopped.

## 2.4 Why doesn't it work?

There are many reasons which can explain why the cavity didn't work:

- We can imagine that the modulation was too weak for me to see it.
- the power amplifier I sometimes used from 2 to  $4 \text{ GHz}$  may have not received enough power, since when i used it again with the reentrant cavity, I found that the power supply i had used couldn't supply enough power ( $4 \text{ A}$  at  $15 \text{ V}$ !).
- A big concern is the size of the antenna, which is only  $1 \text{ mm}$  long, whereas the wavelength of the microwave is around  $10 \text{ cm}$ .
- The copper rusted very fast, after 2 weeks it started to look a bit tired.
- the antenna is in a place where the field is evanescent (outside the crystal), so it may be a place where it is hard to get the  $TE_{101}$  mode.
- most probably, the reason why the cavity didn't work is that we asked for a width of cavity of  $16.89 \text{ mm}$  instead of  $13.89 \text{ mm}$ , which is very strange since the mathematica program gave us the second value. Whether i copied it wrongly or a factor 2 which had to be added entailed an error is not sure. Anyway, even with such a length, the resonant frequency should have been of  $3.2 \text{ GHz}$ , which is in the range i scanned. Nevertheless, the phase matching condition wasn't very well satisfied in this case.

I asked the workshop to build two bigger waveguides so that we could check the importance of the length of the antennas. I designed them so that they should be resonant around  $3 \text{ GHz}$  with a  $TE_{101}$  mode and a  $TE_{102}$  mode. One of them has a height of  $5 \text{ cm}$  and the other one  $1.5 \text{ mm}$  just like our cavity. The lids for the  $TE_{101}$  and  $TE_{102}$  modes have spots for two antennas so that we can look at the transmitted power from one to the other (when the cavity is resonant, the power transmitted should be bigger). Unfortunately, I didn't have enough time to try them.

# Reentrant cavity

Since the rectangular cavity didn't work, we decided to try another way. We had a New Focus cavity in the lab that was resonant at about  $4\text{ GHz}$  (tunable with a screw over roughly  $200\text{ MHz}$ ). I tried to find some documentation about these cavities and the most useful I found was [10]. A reentrant cavity had already been designed in the lab by Yacin Karim, but he used the Sperry curves, which are not very accurate and only provide us with ratios (of radii, heights in the cavity (see fig 3.1 for a picture of a reentrant cavity ))

## 3.1 Principle of a reentrant cavity

### 3.1.1 Presentation

A reentrant cavity is a cylindrical cavity which is fed by an antenna. It is made of a cylindrical conductor in the center which is ended by a gap. The outer wall is a cylindre. A picture of it can be seen on fig 3.1



Figure 3.1: Picture of the reentrant cavity I designed

The crystal is put on the post and has to be as close as possible to the lid, if possible with no gap to allow most of the microwaves inside the crystal. We can see this cavity as a bottom part full of magnetic field and a top with a lot of electric field. The mode is thus excited by a loop antenna in the bottom part of the cavity. The New Focus cavity even has a short circuit as an antenna (the antenna touches the post which is grounded).

### 3.1.2 Reduction factor

According to [10], there is a standing wave in the crystal, which we can consider as two travelling waves. Thus, the eq 1.20 gives us the reduction factor  $\beta$  for a reentrant cavity<sup>1</sup> :

$$\beta = \frac{1}{2} \left| \frac{\sin(u_+)}{u_+} + \frac{\sin(u_-)}{u_-} \right| \quad (3.1)$$

where

$$u_{\pm} = \frac{\pi f_m L}{c} (\sqrt{\epsilon_r} \mp n) \quad (3.2)$$

#### Length of the crystal

It is interesting that the reduction factor is a function of the length of the crystal. Nevertheless, the really important value is the modulation depth  $\eta$  which is a function of  $\beta L$ . I have plotted the value of the modulation depth as a function of the crystal length in fig 3.2 for a modulation frequency of 2 GHz, for reasons we will see later.

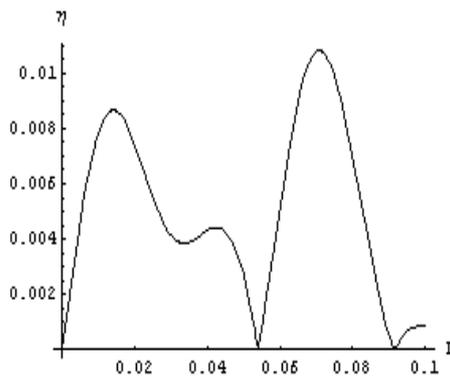


Figure 3.2: Modulation depth (arbitrary units) as a function of the crystal length (in meters...) , assuming a given voltage across the crystal

We must be careful using this plot since it assumes that the voltage applied across the crystal doesn't depend on the crystal length. This is not exactly true, since the voltage depends on the Q factor of the cavity which is a function of the dimensions of the cavity. And we must bear in mind that the dimensions of the cavity must be chosen according to the dimensions of the crystal (the crystal has to fit inside the cavity).

#### Resonant frequency

Since the dimensions of my crystal were given, I also found interesting to plot the modulation depth as a function of the modulation frequency. It can be seen on fig 3.3 that the lower the frequency, the better the modulation depth.

Since last time someone tried to design a reentrant cavity in the lab they failed to see any modulation, I thought it could be a good idea to try and design a cavity with a rather low resonant frequency.

---

<sup>1</sup>This implies that the electric field is along the optic axis of the crystal, which will prove to be a good approximation in the simulations

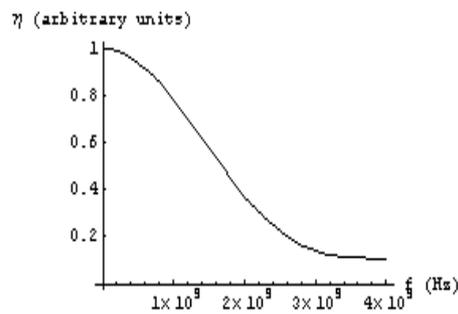


Figure 3.3: Modulation depth as a function of the modulation frequency

It is also interesting for the next attempt to build a microwave phase modulator to order a crystal with dimensions chosen so that the value of  $\eta$  is as big as possible. (Both the dimensions and the resonant frequency are important for the value of  $\eta$ ).

## 3.2 Computer simulations using Superfish

In order to design a reentrant cavity that would allow optical phase modulation, I had to look on the internet for a numerical solver of Maxwell's equations. I found many softwares but the only one i could use was Superfish, which runs on PCs.

### 3.2.1 Superfish

Superfish is a collection of programs developed at Los Alamos National Laboratory which could be found on the server [laacg1.lanl.gov](http://laacg1.lanl.gov) (IP 204.121.24.18, username: SFUSER, password: ftpsuperfish). The package solves Maxwell's equations in two dimensions for both static magnetic and electric fields as well as radio-frequency electromagnetic fields using finite element analysis. The codes analyze a user-defined cavity geometry to generate a triangular mesh that is subsequently used in the finite element analysis of the wave equation. Radio-frequency solvers iterate on the frequency and field calculation until finding a resonant mode of the cavity.

### 3.2.2 Equivalent crystal

The package can only solve Maxwell's equations in two dimensions and thus our cavity must have a cylindrical symmetry (so that we can describe the reentrant cavity with only two coordinates: along the height and the radial distance.).

It is not the case since our crystal is rectangular. But like in [10] we decided to model the reentrant cavity as a section of coaxial transmission line that is short circuited at one end and capacitively loaded by the electro-optic crystal at the other end. Thus, the resonant frequency should not change too much if we replace our actual rectangular crystal with a virtual cylindrical one with a different  $\epsilon_r$  chosen so that the capacitance remains the same.

Using the formula

$$C = \epsilon \frac{S}{e} \quad (3.3)$$

for a parallel plates capacitor (where  $S$  is the surface of the plates and  $e$  the distance between them), we can easily find the equivalent  $\epsilon_r$  of the cylindrical equivalent crystal. We decided to make the inner cylinder radius the size of the length of the crystal: the longer the inner radius, the lower the resonant frequency, and i had difficulties to find a resonant frequency over  $1.5\text{ GHz}$  with realistic dimensions (see 3.2.4).

We can choose the equivalent crystal with the same surface as the actual one or choose its radius to be the length of the actual crystal. As long as we can approximate the reentrant cavity as a coaxial transmission line capacitively loaded, it shouldn't change anything. I tried both of these methods and they give the same results at better than 5%, which is really good compared to the other problems i had.

### 3.2.3 Problems solved

Writing a Superfish program is not very easy and i spent quite a long time trying to model our cavity. It was a better idea to try one step at a time, first with a cylinder, then with a reentrant cavity with no crystal inside and then add the crystal. I also spent a lot of time analyzing examples given in the litterature, but this program is usually used for particle physics and is used in a much more general way than the way I used it.

The first difficult thing was to understand the right order to run each program (double click the .am file, then run cfish and then sfo and then open the .t35 file to see the results) and how to write a correct program for the simplest thing: an empty cavity. Then, I tried to reduce the calculation time (up to 1 hour at the beginning) by reducing the mesh until the results became very different.

#### Empty cavity

Yacin Karim had already studied an empty reentrant cavity, and I used the values he gave in his report [4] to calculate the resonant frequency. Using Superfish simulation (see B.1), i found  $f_{res} = 2655\text{ MHz}$  which is the value Yacin found at better than 1%.

#### Mesh

The first times I used the program, it took it ages to send me an error signal. I increased the size of the mesh by a lot and when i was able to write programs that worked, i decreased its size until I kept having the same resonant frequencies and calculated fields.

#### Indices

We can only give one  $\epsilon_r$  for our crystal, so we used the index along the optic axis, thus assuming that there is almost no radial electric field in the crystal. This approximation is very well respected, as can be seen on the figures 3.4 and 3.5 of the simulations.

### 3.2.4 Design of our cavity

There are a few conditions that needed to be respected for the design of the reentrant cavity:

- first the crystal must fit in, which means that the inner radius must be bigger than the length
- the outside dimensions of the cavity must fit in the optical cavity, and its height must not be too big because the cavity has to stand on an adjustable mount on the optic table

- it must be makeable by the workshop, which means that the difference between the outer radius and the inner radius cannot be too small (and we have to fit an antenna inside anyway)
- it must have a resonant frequency of a few  $GHz$

I ran the superfish program (given in B.2) quite a few times with different dimensions, and it turned out that the resonant frequencies were usually very low (a few hundred of MHz). To increase the frequencies, it would have been easier with a smaller cristal (shorter or less wide). The only way to have a resonant frequency over one GHz was to make the difference between the outer and inner radius rather small ( $5\text{ mm}$ ).

The results of the calculations are shown on fig 3.4, which contains the values we sent to the wokshop (in millimeters). The figure is cylindrically symetric around the vertical axis  $x = 0$

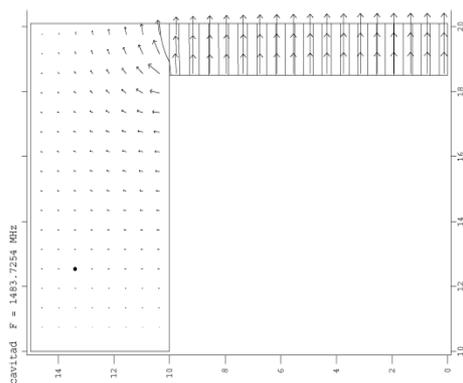


Figure 3.4: Calculated values of the electric field for our reentrant cavity. Resonant frequency : 1484 MHz. The equivalent crystal radius is equal to the length of the actual crystal

We can see that we considered the height of the crystal to be  $1.6\text{ mm}$  instead of  $1.5\text{ mm}$  (which is the value given by the manufacturer) because of the problems encountered with the rectangular cavity.

But this can generate a gap if the crystal is actually  $1.5\text{ mm}$ . We can see the corresponding simulation in fig 3.5. We must consider that the electric field is much stronger in the gap than in the

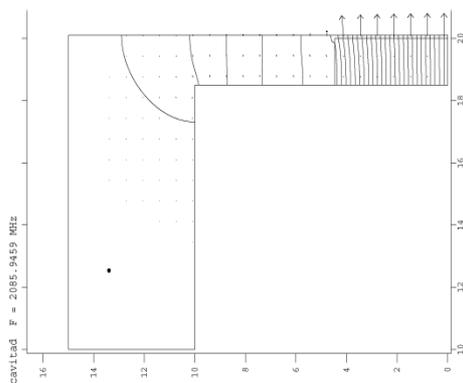


Figure 3.5: Calculated values of the electric field and resonant frequency for our reentrant cavity with a gap. Resonant frequency: 2065  $MHz$

crystal (which can be seen by looking at these values in a tabular, and on the figure when we know

that the size of the arrows is normalized to the values of the electric field at  $r = 0$  by comparing with fig 3.4) . The resonance frequency is also much higher.

We asked the workshop to build the cavity with the values shown in fig 3.6.

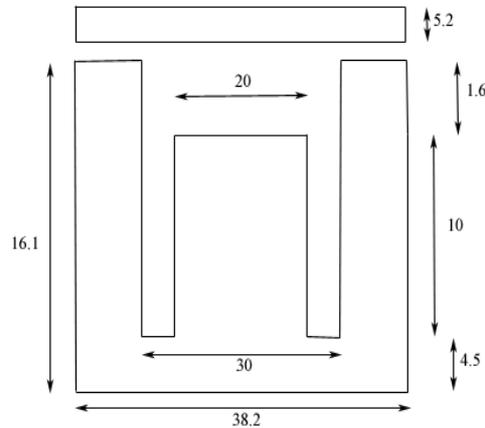


Figure 3.6: Dimensions asked to the workshop

We asked the workshop to drill a square exactly on a diameter (in the outer cylinder) and to place one or two spots for antennas at  $90^\circ$  of this diameter respectively for the first and second cavity. Just like for the rectangular cavity, I asked for a hole for the antenna as small as possible, but this time i had the cavity made out of aluminium and polished the best they could: the Q factor of the cavity depends on the finish of the cavity and when it increases the value of the electric field across the crystal increases for a given input.

### 3.3 Oven

At this time I still aimed at finishing my project so I designed an oven for this cavity. It had to be very easy to build for the workshop so that they could build it quickly. It can be seen in fig 3.7. It is made of two aluminium plates that can be screwed to tighten the cavity in between. Each

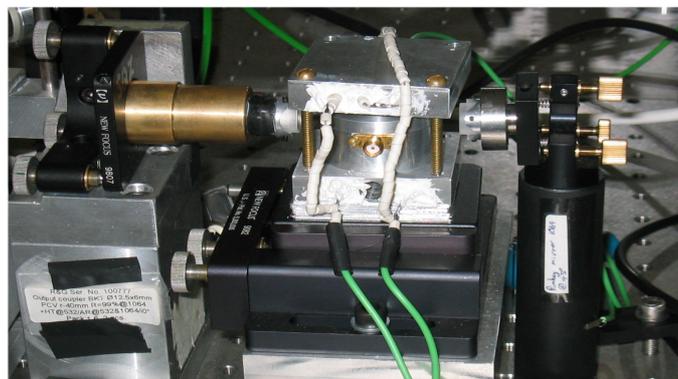


Figure 3.7: Oven for the reentrant cavity

plate has 4 resistors of  $0.82 \Omega$  in series and they are put in parallel. Thus, the equivalent resistor is  $1.64 \Omega$  which is quite close to the ideal  $1.6 \Omega$  of the temperature controller. I put the thermistor in the lid of the reentrant cavity.

I was given an older model of temperature controller and I had decided to leave the crystal at a temperature around  $70^\circ$  (At first I wanted to test the temperature controller, and then thought it would be a good idea to test the modulation at a temperature closer to the final temperature). But the temperature controller went down during the week end, and when i tried to align the crystal, it proved very difficult. It turned out that the crystal was slightly cracked. It is probably due to it cooling down too fast. Some people in the lab think it can still be used since it is only cracked on an outter region, but nobody tried it.

## 3.4 Results

In this section I used the cavity which has two antennas, so that i could easily find the resonance (with a signal generator and a spectrum analyzer).

### 3.4.1 Resonant frequencies

When I tried and test the cavity, I noticed that the crystal inside could move, which meant that this time, the gap was too big for the crystal. Thomas has the idea to put aluminium foils between the crystal and the lid and the post. It was rather difficult to cut aluminium foils without folding them (to avoid stress on the crystal) and as close as possible to the size of the crystal (same reason).

I found that two aluminium foils prevented the crystal from moving inside the cavity, and modulation and transmission (with the two antennas) proved that there was some modulation at  $1.609\text{ GHz}$ . After the crystal cracked, when i changed the crystal i also removed one foil, and then the modulation occured at  $1.702\text{ GHz}$ . Then I removed all the aluminium foils and tried to move as little as possible the cavity so that the crystal would not move inside. The modulation occured at  $1.987\text{ GHz}$ . We must acknowledge that this frequency varies slightly in the time, maybe because of temperature shifts. We decided to keep using no aluminium foils for crystal safety reasons and also because we had a power amplifier in the range 2 to 4  $\text{GHz}$ .

It is interesting to emphasize that the modulation occurs at the same frequency as the resonance of the cavity in transmission, which is very helpful since the Q factor of quality is very high, making the scanning of the cavity very painful (all the more that there is sometimes a lot of pickup).

### 3.4.2 Modulation

As we did for the rectangular cavity, we transformed phase modulation into amplitude modulation thanks to a quarter and a half wave plates. We used a power amplifier after the signal generator with a 30  $\text{dB}$  gain. Its maximum output power was 37  $\text{dBm}$ . On the spectrum analyzer, light modulation looked like fig 3.8.

We can measure the Q factor of our modulator: 300. It is interesting to know that i had measured a Q factor of 4000 for the New Focus cavity at 4  $\text{GHz}$ . Considering that it is the first time i built a reentrant cavity and that our cavity was made of aluminium and polished the best we could but still not very well, it looks like a rather good result.

### 3.4.3 Conclusion

Yacin didn't see any modulation, but I think the reason is mainly because of his crystal, which was almost 3 times as high as ours (electric field divided by three) and rather short ( $\beta L$  for our cavity was 10 times as big as his). In the end, I insisted a lot on the finish of the cavity in the workshop,

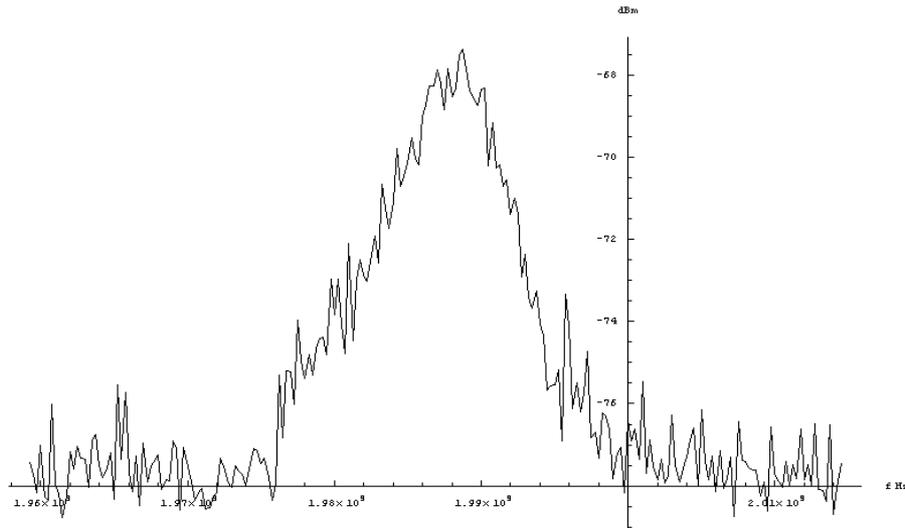


Figure 3.8: Amplitude modulation as seen on the spectrum analyzer

and the gap between the crystal and the lid was much smaller in our cavity. Since I had to feed the cavity with a lot of power to see some modulation, it is not surprising that he couldn't see any.

The first step of the optical frequency comb generation took me a bit more than 4 months to achieve. I had to be independent since it is not the usual field of the quantum optics lab. What was left was the building of the optical cavity around the modulator.

## 4.1 Design of the experimental setup

We use laser beams with a gaussian profile, and ray optics cannot always be used to determine the properties of the beam as a function of its location. For this part, I have mostly used [9].

### 4.1.1 ABCD Matrices

A gaussian beam propagating through a linear medium can be described by its radius of curvature  $R$  and its waist  $w$ . We need to introduce  $q$  defined as

$$\frac{1}{q} = \frac{n}{R} - i \frac{\lambda}{\pi w^2} \quad (4.1)$$

where  $\lambda$  is the wavelength of the optical beam in free space. We can then find  $q$  at any other place using the ABCD matrices:

$$q_2 = \frac{Aq_1 + B}{Cq_1 + D} \quad (4.2)$$

Each lens, mirror and medium has its corresponding ABCD matrix. The global ABCD matrix can be found by multiplying these matrices:  $M_{tot} = M_n * M_{n-1} * \dots * M_1$

For example, for a converging lens or a mirror of radius of curvature  $R$ , we have respectively the matrices:  $\begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}$  and  $\begin{pmatrix} 1 & 0 \\ -2/R & 1 \end{pmatrix}$

### 4.1.2 Mirrors for the optical cavity

We wanted to build a stable mirror resonator with the crystal inside. This is very well explained in Siegmann p747 (ref [9]). The length of the cavity is given by the frequency of our modulator, and we want the middle of the crystal to be on the waist. We also want the waist to be rather small in order to have non linear effects, and the optimal size of the waist was given to me by Thomas. We also had a set of mirrors and I had to choose among them. So I wrote a mathematica program (see A.3) with the radii of curvature of the mirrors, the length of the cavity as inputs, which gave me the distance between the crystal and each mirror and the size of the waist. We can also check the stability of the cavity thanks to the criterion given in [9], being careful that the equivalent length of the cavity is not the optical length but the length given in the mathematica program (the crystal makes the cavity appear smaller than it really is!).

In the end, I chose to use two mirrors with equal radii of curvature:  $R = 40 \text{ mm}$  and 99% reflectivity. The corresponding waist is around  $80 \mu\text{m}$ , but is slightly dependent on the size of the cavity and thus on the modulation frequency.

### 4.1.3 Lenses: mode matching

Knowing the size of the waist and its position (at the centre of the crystal, in the middle of the optical cavity), I needed to find a set of lenses able to produce the right waist in the correct location (ie not too far from the laser). It is very important to mode match the laser to the cavity, otherwise most of the power will be reflected, since only a few modes are stable in the cavity and can go through.

#### Calculations

Knowing which focal length were available for the lenses, I made another mathematica program in order to calculate the different lengths between the lenses. In the end, I chose to put two lenses. All the results of the calculations can be seen in fig 4.1

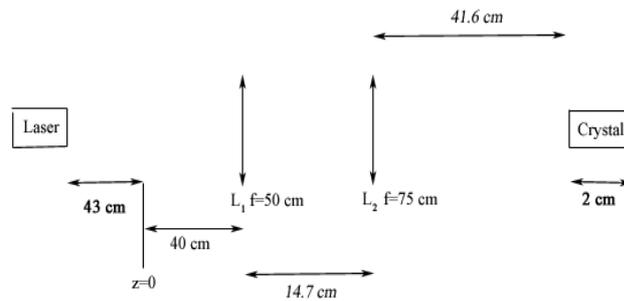


Figure 4.1: Results of the calculations for mode matching the optical cavity: data in bold, results in italic, input with normal letters

The first lens is mounted on a translation device so that we can adjust the waist size and position slightly without having to realign everything.

#### Beam scan

In order to characterize the beam, we need to know its waist size and position. This can be done by using a beam scanner, which gives the size of the beam thanks to a photodiode mounted behind two rotating blades. Taking many measurements at different positions, we can then use a computer program to find the waist size and position.

For our laser after the faraday isolator, we found a waist size of  $135 \mu m$  located at  $-425 mm$  for one of the coordinates and  $153 \mu m$  and  $-434 mm$  for the other transverse direction. This means that there is probably an alignment problem somewhere, but it is a common problem in optics.

For the mode matching, after adjustment of the first lens position, I found that the waist size was  $82 \mu m$ , which is rather satisfying.

## 4.2 Setting up the laser

I have used a YAG laser pumped by a single diode driven at 2.000 A. It delivers in these conditions 1.20 W of continuous infrared light at 1064 nm.

### 4.2.1 Quarter and half wave plates

Quarter and half wave plates are anisotropic media, like our crystal (see chapter 1). They are very useful in optics because they allow to change the polarization of the light. They are usually mounted on a rotating device (in the plane perpendicular to the beam) so that we can modify the polarization as we want. A half wave plate (respectively quarter) induces a  $\pi$  (respectively  $\pi/2$ ) phase shift on one of its axis compared to the other. They work at a given wavelength.

We used these plates for three purposes. The first one was to make the polarization of the laser linear so that we could use it in the Faraday isolator.

The second one was to convert phase modulation to amplitude modulation. A quarter wave plate placed before the phase modulation made the polarization circular. Then the light was phase modulated on one of the axis, and thus was delayed compared to the other. A half wave plate at  $45^\circ$  of the optic axis of the modulator combined with a polarizing beam splitter making another  $45^\circ$  transforms the phase modulation into amplitude modulation (calculations are rather easy).

The third use for these plates is to control the power of the beam in the experiment: after the Faraday isolator, we put a half wave plate followed by a polarizing beam splitter. By turning the half wave plate, the energy of the beam goes progressively in one or the other direction. One beam is stopped, the other goes in the experiment.

### 4.2.2 Faraday isolator

This device prevents the light from going back into the laser, which can make its output unstable and even damage it. Its input must be linearly polarized light. It is made of a first polarizing beam splitter (PBS) which lets all the incoming light from the laser through, then there is a medium which makes the linear polarization change by  $45^\circ$ , always in the same direction, whatever the direction of the light. Then, there is another polarizing beam splitter which lets all the light from the laser through. If some light comes back, the first PBS lets only one polarization go through, it is stirred of  $45^\circ$  like the light of the laser and is all reflected away from the laser by the first PBS.

### 4.2.3 Beam stirrer

The beam stirrer is a device made of two mirrors 4 cm apart, mounted on the same mount. One of the mirror can be moved independently of the other. It allows translation of the beam in a plane transverse to its propagation. The beam can also be tilted slightly. This device is used for fine alignment, everything coarse is usually made before using it.

A schematic of the beginning of the experiment is in fig 4.2.

## 4.3 Alignment

In optics, it is very important to know where the beam is, especially for an invisible laser beam like ours. It is the reason why in the lab every optical objects are placed at a height of 13 cm above the table. It is also important to center as well as possible the lenses and mirrors so that the beam remains in the  $TE_{00}$  mode.

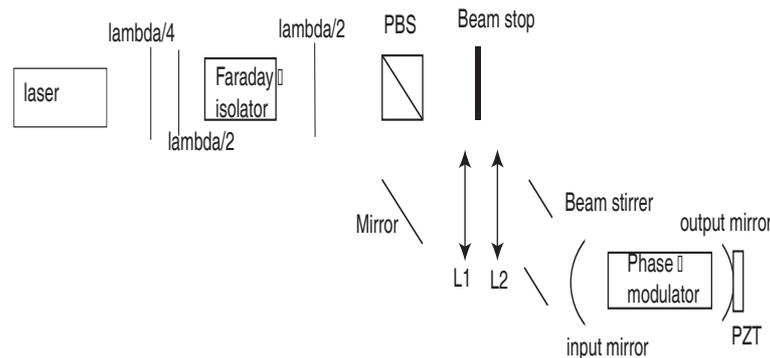


Figure 4.2: Schematic of the first part of the experiment.

### 4.3.1 Mirrors, lenses

A good way to do that is to tighten a screen at the end of the table, as far away as possible from the last object put on the table and to mark where the beam is. We also put an iris but this time just after the last object. Then, we put in the new lens and check that the back reflexion goes back through the iris and that the beam hasn't moved on the screen. Then the lens can be tightened to the table. For mirrors, there is a big and a small reflexion and it is a good idea to try and make them one: usually mirrors are mounted on a stirring device so there are more degrees of freedom than for a lens. It is still necessary to look on the screen to make sure that the beam still goes straight. This can be a rather long process, especially with mirrors. Due to the size of the optical cavity, we had to mount the mirrors on a heavy overhanging piece of metal, which made adjusting the height of the mount really annoying (see fig 3.7)

### 4.3.2 Optic fibers

Another tricky part in optics is to couple the light of the laser in an optic fiber, especially when it is a single mode fiber. There were two types of connector to the mount in my lab, one was the Angled Physic Connector, the other one was not angled. For each one, there is a corresponding fiber coupled laser which i could use. The fiber coupled laser is mounted in the place where the optic fiber will be in the end, facing the big laser beam. The idea is then to align the two laser beams using mirrors. and the mount for the optic fiber. This can be done by choosing two points and checking that both laser beams are at the same place. It is also important to mode match the fiber, and this can be done roughly (which is often enough) by moving lenses trying to make the two laser beams the same size at different points.

### 4.3.3 Cavity

Aligning the cavity was the hardest bit: there are three adjusting screws on each mirror of the optical cavity, five on the mount of the microwave reentrant cavity and five others on the beam stirrer. Since the holes on the microwave cavity are only  $1 \text{ mm}^2$ , it is hard to get the laser beam inside without clipping. It is also hard to put the crystal manually, since even movements that are not visible can make big difference on the amount of light going through. Magnus and I aligned the cavity at least 4 times, putting the elements in different orders (mirrors, microwave cavity and crystal.).

The output mirror is mounted on a PZT (a piezo-electric crystal: when we apply a high voltage

to this crystal, it gets a little bit bigger or smaller, pushing the mirror backward or forward a few wavelengths) and we use it once there is enough light going through the cavity. If we apply a triangular signal to the PZT (from  $-100\text{ V}$  to  $200\text{ V}$  for example), we scan the cavity, which means that we can see the different modes going through the cavity one after the other. By reducing the amplitude of the signal and changing its offset, we can select a given mode. Using an infrared camera, we can see the shapes of the different modes on a TV screen. Our aim is to make the different modes as "circular" as possible by moving the mirrors and the beam stirrer. Pictures of different modes at this stage are given in fig 4.3

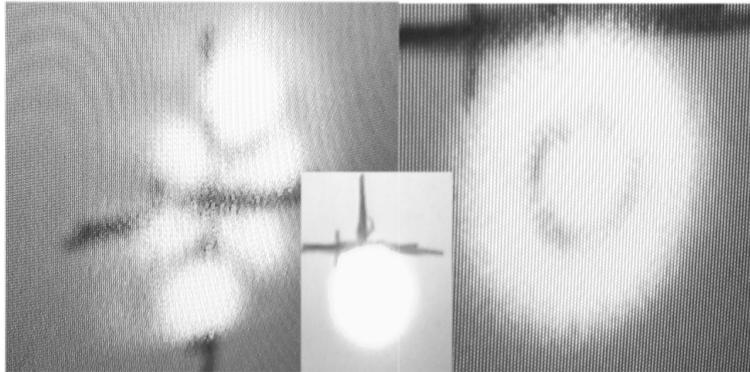


Figure 4.3: Examples of modes that can be seen after the cavity. Left: not aligned. Middle :  $TE_{00}$  mode. Right: doughnut mode.

Once we get good looking modes, we put a photodiode after the cavity and scan it. We can send the signal of the photodiode to an oscilloscope triggered by the triangular signal. The photodiode shows us for which size of the cavity there is a mode going through. The aim is to increase the  $TE_{00}$  mode and decrease the other by adjusting all the screws, until this mode is as big as possible (see fig 4.4)

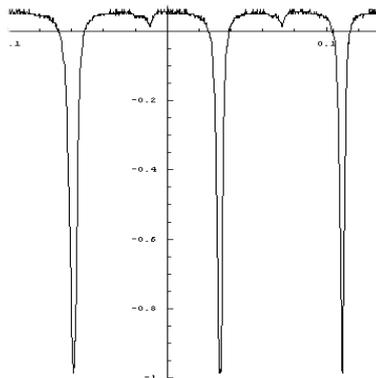


Figure 4.4: Signal sent by the photodiode when the cavity is aligned and scanned

Two other modes can usually be seen, they are the  $TE_{10}$  and  $TE_{20}$  modes. The first one is due to misalignment and the other one to mode matching problems. At this stage, at the beginning, we could see the laser mode hopping which resulted in a very unstable picture on the oscilloscope screen. We solved the problem by changing the temperature of the laser until the problem disappeared. Now, we want to find a way to lock the cavity on the  $TE_{00}$  mode, which means that even when the cavity is moving due to sounds or other vibrations, there is a feedback that compensates these vibrations.

## 4.4 Cavity locking

### 4.4.1 Principle

We used the idea of Pound Drever and Hall. It consists in putting on the PZT on the output mirror a weak triangular signal at around  $200\text{ kHz}$ . This creates a small phase modulation on the light, which can be seen as sidebands. When the cavity is for example just a little bit bigger than its usual size for the  $TE_{00}$  mode, one of the sidebands goes less through the cavity than the other, which results in the apparition of amplitude modulation, which can be detected by the photodiode. Using a demodulator, this gives an error signal that can be sent back to the PZT to correct the size of the cavity.

### 4.4.2 Experimental realization

#### Electronics used

In order to see the error signal on an oscilloscope, we want to send the addition of two signals to the PZT, a low frequency one (to scan the cavity) and a high frequency one (to get the error signal). This will also be useful when we want to add the error signal to the modulation signal to actually lock the cavity. This was realised thanks to a home made box with two inputs (one connected to a capacitor, the other one to an inductance) and one output which is the addition of these signals.

We also needed a demodulator, which is a device which takes two inputs, one being the modulation and the other the signal. The output is the amplitude of the signal at the modulation frequency. The signal and the modulation have to be in phase, so there is an optimization to do, which consists in adding or removing coaxial cable between the modulation signal generator and the demodulator.

Another object is the PID (Proportional Integrator Differentiator) which is used as a filter and to lock the cavity from the error signal. The one I used was home made, with a preintegrator, an adjustable offset (to be able to choose on which mode we want to lock, just like when we scanned the cavity with very little amplitude), and an amplifier.

There were also two high voltage amplifiers that I used. The modulation generator was a very good (SRS) function generator on which we could choose the power delivered. We chose  $10\text{ dBm}$  because the demodulator needed  $7\text{ dBm}$  to work properly (and the signal is divided by two). We adjusted its frequency to reach a resonance of the PZT (bigger error signal).

#### Optimizing the error signal

To get the best error signal as we could, we decided to look at it on the oscilloscope, with the circuit built according to fig 4.5

The second generator has a low frequency so that the photodiode has time to get enough signal (otherwise the signal to noise ratio is very low).

This way, once everything is working properly, we can see the error signal on the oscilloscope corresponding to the apparition of a  $TE_{00}$  mode in the cavity (see fig 4.6). Since the error signal was already rather big (big enough for the cavity to remain locked) and I was running out of time, I didn't spend much time optimizing it. Instead I locked the cavity and tried to find a comb.

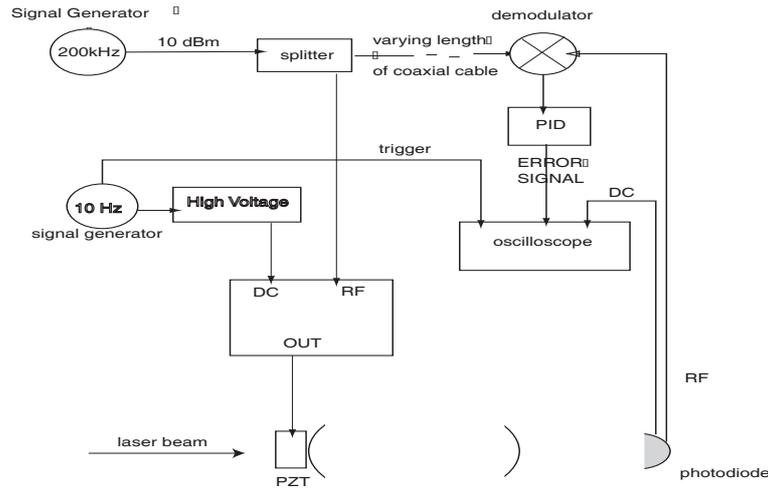


Figure 4.5: How I saw an error signal.

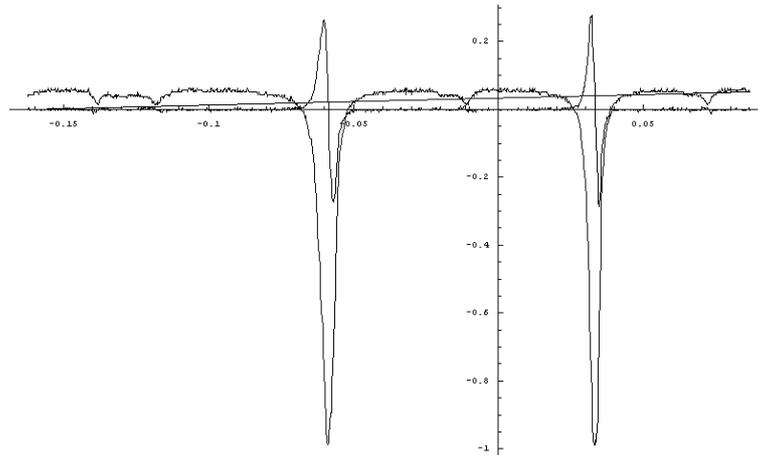


Figure 4.6: Error signal and transmission of the cavity as a function of time while the cavity is scanned

### Locking the cavity

The schematic could be the same as in fig 4.5, except that the error signal doesn't go through the PID and to the oscilloscope but through the home made PID, which output is then amplified with high voltage and sent to the PZT through a coaxial cable replacing the 10 Hz signal.

We adjusted the gain and preintegrator of the PID and the cavity could remain locked for a long time unless someone would bang the door or something like that.

## 4.5 No results

The cavity wouldn't remain locked when I turned the microwave generator and amplifier on, so I tried to look at what was happening when I turned them on while scanning the cavity: the  $TE_{00}$  modes were moving on the oscilloscope, which is probably due to some thermal effect (the crystal gets bigger and the optical length of the cavity is changed). So I locked the cavity after letting the

microwaves on for a while and then the lock worked properly.

I then sent the signal through a poorly coupled optical fiber to an optical spectrum analyzer hoping to see a comb. Unfortunately, its maximum resolution was of about 5 teeth of my comb (10  $GHz$ ), so it didn't mean that there was absolutely no comb.

So I borrowed a confocal cavity that could be scanned (like a spectrum analyzer, with a much better resolution but with no absolute frequency measurement) from the gravity wave group. I optimized the signal and I could see sidebands adding when I turned the microwaves on. This probably meant that i needed to adjust the modulation frequency (in a range of only 10  $MHz$ ) or harder adjust the distance between the two mirrors (which can be done because each mirror has three screws to be adjusted) to meet the condition 1.26. Another way to adjust the modulation frequency, and perhaps the best way in this case, would have been to put a screw in the lid of the reentrant cavity and to screw it in or out to change the resonance frequency of the cavity (by changing the capacitance) . Anyway, before I could try any of these, the crystal moved in its cavity, but i thought something went wrong with the alignment so I lost the alignment trying to get it back. Since I had no time left for the experiment, I stopped there.

I worked very hard on this part of the experiment and Magnus really helped me a lot. We got a lot of advice from Warwick and Andrew and borrowed a lot of equipment from the Gravity Waves and Atom Optics groups. Many other things happened but they would be too long to tell (for example, I repaired a photodiode whose condenser blew, we tried to look at the power in reflection,...).

**Conclusion** I didn't achieve a non linear optical frequency comb. Nevertheless, I made a first step toward it: I designed and tested a reentrant cavity that modulates light at around  $2\text{ GHz}$ . It involved some microwave engineering, which is not the usual field of the Quantum Optics Group. I had to be rather independent during this period. Unfortunately, when I started the optical part of the experiment, it was a really busy time for the physics department, with conferences and the opening of the Centre of Excellence in Quantum Atom Optics, but I was able most of the time to find someone to answer my questions.

This experiment didn't involve very high level physics, but I started it from scratch and thus got acquainted with many useful techniques. It also gave me an outlook on many aspects of research: I learnt a bit of theory ( $\simeq 1$  month), did some numerical simulations ( $1 - 2$  months), designed and tested two types of cavities ( $\simeq 1$  month each) and an oven, and i was in charge of building my experiment ( $1 - 2$  months), which involved alignment and locking, but also suldering and electronics.

This training period gave me a brief overview of the life of a researcher and I really enjoyed it, with the hard times (when I abandoned the rectangular cavity for example) and the great times (when I first saw modulation, when we locked the optical cavity). During all this period, I got help from all the people in my group, but i was also helped a lot by Mel from the Gravitational Waves group and borrowed a lot of equipment from the Atom Optics group. Shane Grieve in the electrical workshop advised me on suldering and wiring and Paul Mc Namara in the mechanical workshop did a very good (and fast) job with the cavities and the oven. Thanks to all these people and all my other "mates", even though I am a bit disappointed i couldn't see an optical frequency comb, I had a really good time in Australia

# Mathematica programs

## A.1 Rectangular cavity

Be careful using this program because the notations are not the ones used in the report.

```

eq1 =  $\beta^2 == \epsilon_r * k_0^2 - k_d^2$ 
eq2 =  $\beta^2 == k_0^2 - k_a^2$ 
eq3 =  $Tan[k_d * l/2] == k_a/k_d * Cot[k_a * a]$ 
eq4 =  $c/nlight == \Omega_{microwavebis}/\beta$ 
eq5 =  $k_0 == \Omega_{microwavebis}/(c)$ 

nlight = 2.23;
c = 3 * 108;
 $\epsilon_r = 28$ ;
l = 0.003;
L = 0.02;
 $\beta = Pi/L$ ;

determinationfrequency = Solve[eq4,  $\Omega_{microwavebis}$ ][[1]]
determinationk0 = Solve[eq5/.determinationfrequency, k0][[1]]
determinationkakb = Solve[eq1, eq2/.determinationk0, k_a, k_d][[4]]
determinationa = Solve[eq3/.determinationkakb, a][[1]]

```

## A.2 Optical length

The input in this program is the frequency  $f_{res}$ . The program calculates the distance between the two mirrors of the optical cavity.

```

eq =  $n * c / (2 * L_{opt}) == f_{res}$ 
c = 3 * 108;
 $f_{res} = 2.095 * 10^9$ ;
n = 2;
 $L_{opt} = L_{cav} + (n_{cris} - 1) * L_{cris}$ ;
 $n_{cris} = 2.23$ ;
( $L_{cris} = 20 * 10^{-3}$ );
Solve[eq,  $L_{cav}$ ]

```

## A.3 Mirrors

This program calculates the waist of the beam as well as the distances between each mirror and the crystal. Its inputs are the radii of curavature of each mirror and the distance between the two

mirrors ( $L_{opt}$ ). It also checks the stability of the cavity.

$$eq1 = 1/R2 == (Lc/(2 * nc) + L2)/((Lc/(2 * nc) + L2)^2 + (Pi * \omega_0^2/\Lambda)^2)$$

$$eq2 = 1/R1 == (Lc/(2 * nc) + L1)/((Lc/(2 * nc) + L1)^2 + (Pi * \omega_0^2/\Lambda)^2)$$

$$eq3 = L1 + L2 + Lc == Lopt$$

$$Lopt = 50.7;$$

$$R1 = 40;$$

$$R2 = 40;$$

$$nc = 2.23;$$

$$\Lambda = 1064 * 10^{-6};$$

$$Lc = 20;$$

$$Solve[eq1, eq2, eq3, \omega_0, L1, L2]$$

$$Leq = 64.6 - 20 + 20/2.21$$

$$g1 = 1 - Leq/R2$$

$$g2 = 1 - Leq/R1$$

$$g1 * g2$$

# Superfish programs

## B.1 Yacin's empty cavity

```

&reg kprob=1, ;set for RF
conv=0.1, ;units in mm
icylin=1, ;cylindrical symmetry
freq=130.91416, ; Starting frequency in MHz
dslope=-1, ; Allow convergence on first iteration
xmin=0.000, xmax=20.000, ymin=0.000, ymax=22.5000,
nbsup=1, ;upper
nbslo=1, ;lower
nbslf=1, ;left
nbsrt=1, ;right
dx=0.5000, ;x increment
dy=0.5000 & ;y increment
xdri=1,ydri=10.5 & ; Drive point location
; problem boundary
&po x=0.000,y=0.000 &
&po x=0.000,y=22.5000 &
&po x=20.000,y=22.5000 &
&po x=20.000,y=0.000 &
&po x=0.000,y=0.000 &
; cylindre simple
&reg mtid=0,mat=0 &
&po x=0.000,y=0.000 &
&po x=0.000,y=7.5000 &
&po x=15.000,y=7.5000 &
&po x=15.000,y=0.000 &
&po x=0.000,y=0.000 &

```

## B.2 Design of our cavity

```

&reg kprob=1, ;set for RF
conv=0.1, ;units in mm
icylin=1, ;cylindrical symmetry
freq=2000.0, ; Starting frequency in MHz
dslope=-1, ; Allow convergence on first iteration
xmin=10.000, xmax=20.1000, ymin=0.000, ymax=15.000,
nbsup=1, ;upper

```

```
nbslo=1, ;lower
nbslf=1, ;left
nbsrt=1, ;right
dx=0.2000, ;x increment
dy=0.200 ;y increment
xdri=12.5,ydri=13.5 & ; Drive point location
; problem boundary
&po x=10.000,y=10.000 &
&po x=10.000,y=15.000 &
&po x=20.1000,y=15.000 &
&po x=20.1000,y=0.000 &
&po x=18.5000,y=0.000 &
&po x=18.5, y=10&
&po x=10,y=10&
&reg mat=2 &
&po x=18.5000,y=0.000&
&po x=18.500,y=4.4700&
&po x=20.000, y=4.4700&
&po x=20.000, y=0.0000&
&po x=18.5000, y=0.0000&
; cristal
&mt epsilon=28,mu=1.000 &
```

## Bibliography

- [1] M. Abramovitz and I.E. Stegun, editors. *Handbook of mathematical functions with formulas, graphs and mathematical tables*, chapter 9, page 361. National Bureau of Standards, tenth edition edition, 1972.
- [2] M.C. Teich B.E.A. Saleh. *Fundamentals of photonics*, chapter 18, pages 697 – 715. Wiley-interscience, 1991.
- [3] Manuel Joffre. Optique non linéaire, cours de dea. <http://www.enseignement.polytechnique.fr/profs/physique/Manuel.Joffre/dea/cours.pdf>.
- [4] Yacin Karim. Realization of an electro-optic modulator, 2001.
- [5] D.Lee L.R. Brothers and N.C Wong. Terahertz optical frequency comb generation and phase locking of an optical parametric oscillator at 665 ghz. *Optics Letters*, 19(4):245–247, February 1994.
- [6] Y.Takeuchi M. Kourogi, B.Widiyatomo and M. Ohtsu. Limit of optical-frequency comb generation due to material dispersion. *IEEE Journal of Quantum Electronics*, 31(12):2120–2125, December 1995.
- [7] Jesse Carl Petersen. A microwave resonant electro optic modulator for optical frequency comb generation, 2002.
- [8] David M. Pozar. *Microwave engineering*. John Wiley and Sons, 2nd edition, 1998.
- [9] Anthony E. Siegman. *Lasers*. University Science books, 1986.
- [10] Mark D. Skeldon. Microwave phase modulators for smoothing by spectral dispersion. *LLE Review*, 68:192–208, 1996.