Fission fragment angular distributions for the systems $^{14}$N+$^{232}$Th and $^{11}$B+$^{235}$U at near and sub-barrier energies

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Fission fragment angular distributions have been measured for the systems $^{11}$B+$^{235}$U and $^{14}$N+$^{232}$Th, at sub and near-barrier energies. Complete and incomplete momentum transfer fission components have been separated around 90° c.m. for $^{14}$N+$^{232}$Th. The contribution of incomplete momentum transfer events is found to be about 8% over the present energy range of investigation. In the $^{14}$N+$^{232}$Th case, anisotropy data are found to be anomalous throughout the energy range, where as corresponding data for $^{11}$B+$^{235}$U are found to be normal, with respect to the saddle-point statistical model predictions. The large ground state spin values of $^{11}$B and $^{235}$U are found to play a major role at the sub-barrier region in influencing the anisotropy values.

I. INTRODUCTION

The study of heavy-ion induced fusion-fission reactions has been an active field for many years [1–5]. One of the primary goals of such studies is towards the formation of superheavy nuclei. For successful production of new superheavy nuclei, one needs to understand the dynamical aspects of the fission process [6–8]. Fission fragment angular distribution is a rich source of information on the fission process in general and the dynamics in particular. Early experimental results of fission fragment angular distribution in light ion induced fission were consistent with the standard statistical saddle-point model (SSPM) [9]. In this model, the distribution of $K$ states (the projection of total angular momentum vector, $J\hbar$, onto the fissioning axis, $K\hbar$) for the fission process is determined by the moment of inertia and the nuclear temperature at the fission saddle point. However, in recent years, heavy ion induced fission fragment angular distribution measurements performed from below to above barrier energies have generated much interest due to the failure of the predictions of the SSPM for heavy ion induced fission of actinide targets [1,2,5].

At above barrier energies, nonequilibrium fission (pre-equilibrium, quasifission, and fast-fission) was thought to be a probable cause of this anomaly. According to the preequilibrium fission model [10], the final $K$ distribution for fissioning nuclei is given by $P_f(K) = P_{\text{init}}(K)P_{\text{saddle}}(K)$, where $P_{\text{init}}(K)$ is the $K$ distribution for the initial di-nuclear complex and $P_{\text{saddle}}(K)$ is the Gaussian $K$ distribution at the saddle point. On the whole, the final $K$ distribution is governed by the initial narrow $K$ distribution populated in the formation phase. Experimental signature for pre-equilibrium fission was demonstrated in a series of experiments involving $^3$Be, $^{10,11}$B, $^{12}$C, and $^{16}$O projectiles interacting with $^{232}$Th, $^{237}$Np, and $^{232,238}$U targets [11–14]. As per the pre-equilibrium fission model, anomalous fragment anisotropies appear only for the systems with entrance channel mass asymmetry $\alpha=(A_p-A_t)/(A_p+A_t)$, smaller than the Businaro-Gallone (BG) mass asymmetry $\alpha_{BG}$. For the reactions where the entrance channel corresponds to the case $\alpha > \alpha_{BG}$, as the pre-equilibrium fission component is absent, the measured anisotropies are in agreement with the SSPM predictions.

It has been observed that at sub-barrier energies all the systems with actinide targets show anomalous anisotropies of varying extent with respect to the SSPM. In order to explain such behavior at sub-barrier energies, a few models have been well recognized. The first is the orientation dependent quasifission model [15,16], which has successfully explained the fission fragment anisotropy data of $^{16}$O fusing with a prolate deformed $^{238}$U target. In this model, it has been assumed that during the fusion process, if the angle between the target symmetry axis and the line joining the center of the interacting nuclei, $\theta$, is larger than a critical angle ($\theta_{\text{crit}}$), normal fusion-fission occurs and the angular distribution follows the SSPM. On the other hand, when $\theta < \theta_{\text{crit}}$, quasifission is assumed to occur, and this leads to a large anisotropy because of entrance channel memory. Here $\theta_{\text{crit}}$ is taken to be the angle which best reproduces the anisotropy data. The second model is an improvement over this with the incorporation of spins of target and projectile

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In recent studies it has been shown that the presence of large ground state spins of target [18] and projectile [19] have an influence in the deviation of fission fragment angular distribution from the SSPM at sub-barrier energies. These results have been interpreted in the framework of the entrance channel dependent K-state model (ECD-K) [17]. In this model, if the target and/or projectile have nonzero ground state spin ($I_0$), the entrance channel K-state distribution, instead of peaking at zero, peaks at $K = \pm I_0$. This changes the final K distribution and is shown to be responsible for the lowering of anisotropies at sub-barrier energies.

In our earlier work [20] fission fragment anisotropies at above barrier energies for two different entrance channels, $^{11}$B+$^{235}$U and $^{14}$N+$^{232}$Th, leading to the same compound nucleus ($^{246}$Bk), and at similar excitation energies viz., 40–60 MeV, have been reported. It was observed that even though $\langle f^2 \rangle$ obtained from Wong model [21] calculation, which fits the fusion excitation function of $^{14}$N+$^{232}$Th is lower than that for $^{11}$B+$^{235}$U, measured anisotropy for $^{14}$N+$^{232}$Th is found to be larger than that for $^{11}$B+$^{235}$U, indicating an entrance channel effect. Anisotropy values of $^{11}$B+$^{235}$U were found to be normal, whereas those for $^{14}$N+$^{232}$Th were found to be anomalous with respect to the SSPM. It may be noted that in $^{14}$N+$^{232}$Th and $^{11}$B+$^{235}$U, while the target $^{232}$Th has ground state spin zero, $^{235}$U has a ground state spin 7/2. The projectiles, $^{11}$B and $^{14}$N, have ground state spins of 3/2 and 1, respectively. It is interesting to see the effects of both the entrance channel and the ground state spins in the fission of the above two systems. The purpose of the present paper is to give more details of the experiment and analysis reported in Ref. [20], and also to extend this study to sub-barrier energies, not reported earlier.

The paper is organized as follows: Sec. II presents the experimental setup and the measurement procedure, while Sec. III describes the data reduction techniques and the experimental results. The SSPM model calculations are discussed in Sec. IV. In Sec. V, discussion of the results, for both above and below barrier energy regions and the theoretical model calculations, are presented. Finally, the conclusions are given in Sec. VI.

### II. EXPERIMENT

The compound system formed from the reactions of the present investigation ($^{14}$N+$^{232}$Th and $^{11}$B+$^{235}$U) is highly fissile and decays nearly totally by fission. Hence fusion cross sections for these systems are assumed to be the same as the fission cross sections. The fission fragment angular distribution measurements for the $^{14}$N+$^{232}$Th system were carried out at 15UD Pelletron accelerator of Nuclear Science Centre (NSC), New Delhi, the fission data for $^{11}$B+$^{235}$U were either partially or totally depleted with an active area of 600 $\mu$m$^2$ aluminum foil. The fission fragment angular distributions were measured by employing $E-\Delta E$ silicon surface barrier telescopes. $\Delta E$ detectors are fully depleted silicon detectors having an active area of 70–100 $mm^2$ and of thickness 10–12 $\mu$m. The E detectors are either fully depleted or partially depleted silicon detectors having an active area of 150 $mm^2$ and a thickness of 300 $\mu$m. A collimator was put in front of each telescope to limit the angular spread in the surface barrier detectors. These telescopes were kept at a distance of 13–15 cm from the target position, inside a scattering chamber, on a movable platform. Solid angles of these telescopes were calculated from the aperture area and the distance of the telescope from the target position. Relative solid angles of the telescopes were also obtained by measurements at overlapping angles.

For the determination of absolute fission cross section, the measured differential cross section was normalized by Rutherford scattering cross section at forward angles. Two monitor detectors were placed in the reaction plane at $\pm 10^\circ$ with respect to the beam. These detectors were mounted on a circular holder permanently fixed to the wall of the scattering chamber to monitor count rates during the experiment. A circular collimator of 1.5 mm diameter was put in front of these monitor detectors to avoid radiation damage due to high count rate of Rutherford scattered particles in the forward angles, during the available beam time. These detectors were either partially or totally depleted with an active area of

<table>
<thead>
<tr>
<th>System</th>
<th>$E_{lab}$ (MeV)$^a$</th>
<th>$\sigma_{\text{fission}}$ (mb)</th>
<th>Anisotropy</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{14}$N+$^{232}$Th</td>
<td>91.6$^b$</td>
<td>823±40</td>
<td>2.06±0.08</td>
</tr>
<tr>
<td></td>
<td>87.5</td>
<td>650±26</td>
<td>1.98±0.06</td>
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<tr>
<td></td>
<td>84.7</td>
<td>518±24</td>
<td>1.9±0.06</td>
</tr>
<tr>
<td></td>
<td>82.7</td>
<td>380±53</td>
<td>1.79±0.08</td>
</tr>
<tr>
<td></td>
<td>80.7</td>
<td>290±27</td>
<td>1.78±0.04</td>
</tr>
<tr>
<td></td>
<td>78.7</td>
<td>214±11</td>
<td>1.6±0.09</td>
</tr>
<tr>
<td></td>
<td>76.6</td>
<td>111±07</td>
<td>1.68±0.08</td>
</tr>
<tr>
<td></td>
<td>74.6</td>
<td>54±04</td>
<td>1.59±0.08</td>
</tr>
<tr>
<td></td>
<td>72.6$^b$</td>
<td>20.3±2</td>
<td>1.51±0.07</td>
</tr>
<tr>
<td>$^{11}$B+$^{235}$U</td>
<td>73.9</td>
<td>1440±138</td>
<td></td>
</tr>
<tr>
<td></td>
<td>71.9</td>
<td>1390±128</td>
<td>1.62±0.06</td>
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<tr>
<td></td>
<td>69.9</td>
<td>1200±112</td>
<td>1.68±0.08</td>
</tr>
<tr>
<td></td>
<td>67.9</td>
<td>1030±72</td>
<td>1.58±0.06</td>
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<tr>
<td></td>
<td>65.9</td>
<td>960±95</td>
<td>1.53±0.08</td>
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<td></td>
<td>63.9</td>
<td>772±54</td>
<td>1.53±0.08</td>
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<td></td>
<td>61.9</td>
<td>630±40</td>
<td>1.21±0.07</td>
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<td>59.9</td>
<td>465±33</td>
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<td></td>
<td>57.9</td>
<td>270±20</td>
<td>1.18±0.06</td>
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<tr>
<td></td>
<td>55.9</td>
<td>132±14</td>
<td>1.15±0.06</td>
</tr>
<tr>
<td></td>
<td>53.9</td>
<td>54.6±5.3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>51.30</td>
<td>7.00±2.0</td>
<td></td>
</tr>
</tbody>
</table>

$^a$Corrected for energy loss in the target.

$^b$Measurements were made with 1.8 mg/cm$^2$ thorium target.
50 mm² and a thickness of 300 μm, which was sufficient to stop all elastic products from the reaction. These two detectors also helped to correct offset of the beam by equalizing the Rutherford rates in the two detectors by properly steering the beam along the horizontal axis. In addition, a third monitor detector was placed at 30° to the beam on the other side of the telescope. This detector was partially depleted with an active area of 100 mm² and a sensitive thickness of 300 μm. The aperture for this detector was circular and kept at 45 cm from the target position. Cross sections obtained using either of the two sets of monitors agree with each other. In the ΔE spectrum, the fission events were clearly separated from the rest of the reaction products.

To check for the amount of transfer induced fission component in the 14N+232Th reaction, folding angle distribution of the fragments was measured by keeping one of the telescopes at 90° in c.m., and using a large area silicon strip detector on the other side of the beam axis to detect the complementary fragments. The silicon strip detector used in the experiment had 12 strips each having 3 mm width, 50 mm height, and 300 μm thickness. This thickness was sufficient to stop all reaction products of interest. Typical resolution of this detector for alpha particles from a Pu source was 60 keV. Each strip gives the position of the detected particles. This detector was kept at a distance of 17 cm from the target position and the angular separation between successive strips was about 1°. The total horizontal angle covered by this detector was around 11°. During the measurements, one telescope was kept fixed at 90° in c.m. and folding angle distribution was measured by appropriately moving the strip detector in the horizontal plane to detect the complementary fragments. In Fig. 1, the two-dimensional spectrum displaying all coincident events in the telescope and the strip detector is shown. In Fig. 2, the one-dimensional position spectrum in the strip detector for coincident fission events is shown.

FIG. 1. Two-dimensional scatter plot of energy of primary fragments (ΔE detector) vs energy of the complementary fragments (strip detector). The events within the gate are the coincident fission events.

FIG. 2. Position spectrum obtained for 87.74 MeV 14N+232Th folding angle measurements. Each peak corresponds to a particular correlation angle of the fission fragments.

III. DATA ANALYSIS

A. Differential cross section analysis

The differential fission cross sections were calculated, using the relation

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{fus/fiss}}(\theta_{\text{c.m.}}) = \frac{1}{2} \frac{Y_{F}}{Y_{M}} \left(\frac{d\sigma}{d\Omega}\right)_{\text{Ruth}}(\theta_{\text{lab}}) \frac{\Omega_{M}}{\Omega_{F}}$$

where Y’s and Ω’s are the yields and the solid angles of fission (F) and monitor (M) detectors; and J is the Jacobian used for the conversion of the laboratory to the center of mass system. Differential cross section of elastic events was calculated from the known angle and energy of the beam assuming all the elastic events to be Rutherford at this forward angle. The measured differential fission cross sections were transformed to the center of mass system assuming symmetric mass division and using Viola systematics [22] for the total fragment kinetic energy.

B. Cross section estimation

The experimental differential cross sections were fitted with the exact expression of W(θ) [9,23], from the saddle-point model. Full angular distributions were measured for 14N+232Th at nine different energies, spanning the energy range 72–91 MeV. Fitted W(θ) were integrated to get the total cross sections at these energies.

For 11B+235U, the angular distributions were measured at six energies spanning the entire energy region from 52 to 72 MeV. Saddle-point model expression of W(θ) was used to fit the angular distribution data and the anisotropy values were extracted. Cross sections at other energies in 11B+235U were determined by measuring the fission cross section at one particular angle and employing the procedure, as discussed in Ref. [24]. It has been shown in Ref. [24] that there is little change in the shape of the angular distribution curve over the barrier region. Therefore the ratio of the total cross section to the differential cross section remains con-
stant or varies slowly over this entire energy domain. On the basis of these results, it is clear that, if the differential cross section is measured only at this crossover angle, it is possible to determine total cross section. In Fig. 3, plots of $\sigma(\text{total})/W(\theta)$ vs $\theta_{\text{c.m.}}$ were shown for three energies. A unique crossover angle is seen around $\theta \approx 130^\circ$.

A plot of the ratio of total cross sections to differential cross sections at the crossover angle as a function of bombarding energy was made for energies where complete angular distribution data were measured. For the other energies, knowing the ratio from the above plot and measuring cross sections at crossover angle only, the corresponding total cross section values were determined.

C. Folding angle analysis

The technique of measuring the angle between the two fission fragments viz., the folding angle, has been found to be an efficient tool for the determination of the presence of incomplete momentum transfer events. At very forward angles, it is not always possible to separate the full momentum and the incomplete momentum transfer components unambiguously. Kinematically, $90^\circ$ in c.m. is the best possible angle to separate the above two components. Folding angle distributions for two typical beam energies, 87.74 and 76.64 MeV, are shown in Fig. 4. The main peak of the measured folding angle distribution is found to be at $\approx 0.8^\circ$ of the angle expected for the full momentum transfer symmetric fission events. The asymmetry in folding angle distribution is observed at about a factor of 8 to 10 below the full momentum component. In the analysis, folding angle distributions were fitted by two Gaussians having the same widths. This width, upon fitting the experimental folding angle distribution, was found to be $\approx 6^\circ$. If we assume the asymmetric component to be due to transfer induced fission, this component is estimated to be around 8% of the full momentum transfer events. This result is consistent with the measurements of the folding angle distribution of Ref. [25].

IV. STATISTICAL SADDLE-POINT MODEL CALCULATION

The measured fission fragment anisotropies were compared with the predictions of the statistical saddle-point model (SSPM). The SSPM calculations were performed using the exact expression of $W(\theta)$ as described in Ref. [9]. This exact expression can also be expressed through an approximate relation,

$$ A \approx 1 + \frac{\langle \ell^2 \rangle}{4K_\alpha}, $$

$$ K_\alpha = \frac{T J_{\text{eff}}}{\hbar^2}, $$

where $T$ is the temperature and $J_{\text{eff}}$ is the effective moment of inertia at the saddle point. $\langle \ell^2 \rangle$ is the mean square angular
momentum of the fissioning system. The saddle-point temperature is given by the expression

\[ T = \sqrt{\frac{E^*_s}{a_f}}, \]

where \( E^*_s \) is the excitation energy of the fissioning system and \( a_f \) is the nuclear level density parameter at the saddle point. \( E^*_s \) can be expressed in the form

\[ E^*_s = E^* - B_f(\ell) - E_{\text{rot}}(\ell) - \Delta E_n, \]

where \( E^* \) is the excitation energy of the compound nucleus, \( B_f(\ell) \) and \( E_{\text{rot}}(\ell) \) are the \( \ell \) dependent fission barrier and rotational energy, respectively. \( \Delta E_n \) is the reduction in excitation energy of the fissioning nucleus by evaporation of neutrons. The effective moment of inertia \( (J_{\text{eff}}) \) is defined as

\[ \frac{1}{J_{\text{eff}}} = \frac{1}{J_x} - \frac{1}{J_z}, \]

where \( J_x \) and \( J_z \) are, respectively, the moments of inertia about an axis parallel to the symmetry axis and perpendicular to it. Inputs to SSPM, \( J_{\text{eff}} \) (effective moment of inertia), the \( \ell \) dependent fission barrier \( B_f(\ell) \), and rotational energy \( E_{\text{rot}}(\ell) \) were calculated using the rotating finite range model (RFRM) [26].

Mean square angular momentum \( \langle \ell^2 \rangle \) is related to the fusion cross section. This quantity can be effectively determined by any fusion model which adequately reproduces the experimental fusion excitation function. There are different prescriptions available for the calculation of \( \langle \ell^2 \rangle \), notable among them are models due to Esbensen (zero point vibrational model) [27], the optical model approach of Udagawa [28], the coupled channel model which takes into account the coupling to various inelastic and transfer channels [29], and sophisticated model of multidimensional tunneling which takes into account radial separation and neck formation degrees of freedom [30]. However, for deformed actinide targets, the Wong model [21] which incorporates ground state static deformation in the calculation of barrier parameters is best suited for fitting of the fusion or fission excitation function. In the present calculation we have followed the Wong model for the extraction of \( \langle \ell^2 \rangle \) values. The barrier parameters, used in the present investigation are the same as used in our earlier work [20]. Fittings of experimental cross section data with the Wong model are shown in Figs. 5 and 6. (The data are presented as a function of \( E_{\text{c.m.}}/V_B \), where \( V_B \) are the respective nominal fusion barriers for the systems.)

In the SSPM calculation a level density parameter of \( \Lambda/9 \) was used. The presaddle neutron numbers were calculated from prefission data of Saxena et al. [31]. Starting from the prefission neutron values, the presaddle neutron values have been obtained incorporating the estimates for the formation, the presaddle, and the postsaddle phases. These values of \( v_{\text{pre}} \) have been extrapolated to higher and lower energies where the required data are not available. For \( ^{11}\text{B}+^{235}\text{U} \), the prefission neutron values of \( ^{11}\text{B}+^{237}\text{Np} \) were taken. For \( ^{14}\text{N}+^{232}\text{Th} \), the average of the values for neutron multiplicities, for \( ^{18}\text{O}+^{232}\text{Th} \) and \( ^{11}\text{B}+^{237}\text{Np} \), were used. In Figs. 7 and 8, the anisotropy values for both the systems, with respect to SSPM, are plotted as a function of \( E_{\text{c.m.}}/V_B \). Experimental results are also summarized in Table I.

V. DISCUSSION

For proper interpretation of anisotropy data in the energy region of our interest, it is better to divide them into two parts. In the high energy part, barrier dependent effects can

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FIG. 5. B+U fission cross sections \( \sigma_{\text{fusion}} \) as a function of \( E_{\text{c.m.}}/V_B \). \( ^{11}\text{B}+^{235}\text{U} \): filled circles; \( ^{11}\text{B}+^{238}\text{U} \): filled squares [39] and open triangles [12]. The solid curve is the Wong model calculation as mentioned in text.

FIG. 6. X+Th fission cross sections \( \sigma_{\text{fusion}} \) as a function of \( E_{\text{c.m.}}/V_B \). \( ^{14}\text{N}+^{232}\text{Th} \): filled circles; \( ^{16}\text{O}+^{232}\text{Th} \): filled down triangles [11]; and \( ^{12}\text{C}+^{232}\text{Th} \): open triangles [13].
be ignored, whereas in the near-barrier region, barrier dependent effects will have influence on the anisotropy. From the systematics shown in Ref. [25], it is evident that, for fission induced by projectiles from $^{12}$C to $^{16}$O on various actinide targets, incomplete momentum transfer events vary between 5% and 15% in the energy range of investigation. Further, for the folding angle measurements in the sub-barrier region by Lestone et al. [32] and Mein et al. [33] a value of $\leq 5\%$ was found, which is consistent with the present folding angle measurements for the $^{14}$N+$^{232}$Th system. Therefore, in the analysis only inclusive measurements for both the systems were taken.

A. At higher energies, $E_{\text{c.m.}}/V_B > 1$

In our earlier work [20] it was pointed out that, at above barrier energies, anisotropy data measured for the above two systems, at similar excitation energies, were found to be different. It is interesting to carry out a similar kind of analysis for different projectiles on a $^{232}$Th target. The SSPM calculation, as described above, was performed for $^{10}$B, $^{12}$C, $^{14}$N, and $^{16}$O projectiles on a $^{232}$Th target. The calculations are shown in Fig. 9. It has been found that anisotropy data for $^{10}$B, $^{12}$C induced fission are normal with respect to the SSPM, whereas the corresponding data for $^{14}$N and $^{16}$O induced reactions are found to be anomalous. This is similar to the observation reported earlier [1], where it was shown that $^{12}$C is the transition point from normal to anomalous nature of fission anisotropies. It is interesting to note that the mass asymmetry value for the system $^{14}$N+$^{232}$Th ($\alpha = 0.886$) is just below the Businaro-Gallone critical mass asymmetry value ($\alpha_{BG} = 0.893$). This transition from normal to anomalous is consistent with the earlier proposed preequilibrium fission model [10].

B. At energies $E_{\text{c.m.}}/V_B \leq 1$

It is generally observed that at sub-barrier energies, irrespective of the entrance channel mass asymmetry value, the fission anisotropies for actinide targets are in general anomalous with respect to the SSPM prediction [1]. However, anisotropy values for $^{11}$B+$^{235}$U are found to be normal, whereas those for $^{14}$N+$^{232}$Th are found to be anomalous with respect to the SSPM. For a better understanding of this normal behavior of the $^{11}$B+$^{235}$U reaction, a calculation based on the entrance channel $K$-state distribution, with spins of target and projectile, is carried out. In the literature, this...
The dashed line is the calculation based on the ECD-K entrance channel involving zero spin of target. It is broadened with time due to coupling between intrinsic distribution in the entrance channel and this initial distribution. The main features of this model are that immediately after fusion the system has a K-state distribution calculation. The main features of this model are immediately after fusion the system has a K-state distribution in the entrance channel and this initial distribution is broadened with time due to coupling between intrinsic and collective rotational degrees of freedom. For reactions involving zero spin of target (and/or projectile), at above barrier energies, interaction at all possible orientations are possible and it leads to a uniform K distribution for each J. At sub-barrier energies, when the projectile interacts preferentially with the tips of the prolate target nuclei, the entrance channel K-distribution, for each J, strongly peaks at K=0. However, if the target and/or projectile has a spin (I₀), the entrance channel K-state distribution, instead of peaking at K=0, peaks at K=±I₀. This is responsible for the lowering of anisotropy at sub-barrier energies. A detailed calculation is performed following the procedure of Refs. [18,19].

Figure 7 shows the results of the calculation for 11B + 235U. The dashed line is the calculation based on the ECD-K state model without incorporating any spin, the dotted-line is the calculation with the target spin of 7/2, and the solid line is a calculation including both target and projectile spins. The dot-dashed line in the figure is the SSPM calculation. The SSPM calculation is found to be more or less the same as the ECD-K state model calculation, with both target and projectile spins. At sub-barrier energies, ECD-K state model calculation is found to be very sensitive to the spins of the target and the projectile and brings down the anisotropy near the SSPM value. In general, the experimental fragment anisotropies for the 11B + 235U system are consistent with the results of a calculation based on the entrance channel K-states model, with inclusion of ground state spins of the target and the projectile. This result supports the fact that at sub-barrier energies, the anomalous fragment anisotropies seen for actinide target systems get washed out with large ground state target or projectile spin (or both). Interestingly, similar results have been observed in the recent fission angular distributions measurements of 10B (spin=3+), 11B (spin =3/2−)+237Np (spin=5/2+) reactions [35].

In the 14N + 237Np reaction, since the ground state spin of 14N is 1+, it is interesting to perform a similar calculation, based on the ECD-K state model, both with and without the spin. Figure 9 represents the calculation with the ECD-K state model. The dotted line represents a calculation with spin zero and the solid line with spin-1. The dot-dashed line is the SSPM calculation as mentioned before. It is clearly revealed that the model fails to reproduce the anisotropy data, both below and above the barrier. This implies that the ECD-K state model alone is not sufficient to explain the data in the entire energy region.

To understand the anomalous nature of 14N + 237Th anisotropy data, an orientation dependent quasifission model [15,16] was next tried. For this purpose we have divided the fission events into two components. The first component is due to the fusion of the projectile with an angle greater than a certain critical angle θcrit with respect to the symmetry axis of the deformed target. The cross section for this component is assumed to lead to normal fusion-fission. The second component is due to fission events at θ < θcrit. Fission from this component is assumed to lead to quasifission. The cross sections for both the components are determined using the simplified coupled channel code CCDEF [34]. The calculated anisotropy is defined as

$$A_{calc} = \left( \frac{\sigma_1}{\sigma_{Tot}} \right) A_1 + \left( \frac{\sigma_2}{\sigma_{Tot}} \right) A_2,$$

where A₁ and A₂ are the anisotropy values for the two components, respectively. The corresponding cross sections are $\sigma_1$ and $\sigma_2$. $\sigma_{Tot}$ is the total calculated fission cross section. For component A₁, the anisotropy values have been calculated using the saddle-point model. The RFRM prescription [26] has been used to obtain ℓ dependent Bₗ, $E_{Tot}$ and $I_{eff}$ values. The presaddle neutron corrections were determined as described previously in the text. For the component A₂ (quasifission), the anisotropy is assumed to increase with beam energy using a linear relation in such a way, which is consistent with the anisotropy values at lower energies (shown as a dotted line in Fig. 10). This assumption of an increasing angular anisotropy with beam energy is consistent with the earlier reported work on 12C + 232Th [33] and the observation of Toke et al. [36] and Back et al. [37] for a series of reactions, where quasifission is dominant. The angle
\[ u = u_{\text{crit}} \] was determined for the angle which best reproduces the anisotropy data. At that particular critical angle, a sharp transition between quasifission and fusion-fission is assumed to occur. For the \( ^{14}\text{N} + ^{232}\text{Th} \), the value of \( u_{\text{crit}} \) is found to be 27°, which is consistent with the \( ^{16}\text{O} + ^{238}\text{U} \) work of Hinde et al. [16], where this angle was reported to be 35°. Mein et al. [33] in \( ^{12}\text{C} + ^{232}\text{Th} \) work have reported this \( u_{\text{crit}} \) to be 18°. Finally, the average anisotropy was determined by weighting the anisotropy with the cross sections for both the components. The results of this calculation are shown in Fig. 10. The solid line represents the quasifission model calculation with \( u_{\text{crit}} \) of 27°. The effect of ±5° change in angle on the calculated anisotropy values is indicated by a dashed line.

This implies that the ECD-K state model alone is not sufficient to explain the data. Orientation dependent quasifission is also required to explain the anisotropy. This can be considered as an indication of a dynamical effect with respect to entrance channel mass asymmetry. While it is observed that a significant amount of quasifission contribution is required to explain the \( ^{14}\text{N} + ^{232}\text{Th} \) data, the \( ^{11}\text{B} + ^{235}\text{U} \) data could be described well without the need for quasifission contribution.

C. Analysis by preequilibrium fission model

The concept of preequilibrium fission [10] is distinct from quasifission (QF) and fastfission (FF). According to this model, in the heavy ion induced fusion-fission process, in addition to regular compound-nuclear (CN) fission events, a significant fraction of noncompound nuclear fission like events due to a preequilibrium (PEQ) mechanism will be present. The PEQ events originate during the \( K \) equilibration of the composite system. Depending on the entrance channel asymmetry, if the injection point is to the right of the Businaro-Gallone (BG) critical point of mass asymmetry \( s_{\text{a,a}} \), the composite system evolves with increasing values of mass asymmetry, the driving force favors amalgamation of the two interacting nuclei which results in the fusion-fission process. However, if the entry point is to the left of the BG point, the driving force is such that the system develops into smaller asymmetry values, while the smaller partner gains in mass at the expense of the later. The dinuclear system in this case may reseparate as fissionlike events without \( K \)-equilibration and formation of the compound nucleus. The characteristic signature of these events will be their low values of the \( K_0 \) and the resultant large values of A. This model, as originally proposed [10], was able to explain a large body of anisotropy data at above barrier energies [11–13].

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In a recent work, Thomas et al. [38] have extended the above model to sub-barrier energies. It has been shown that when fusion occurs at relatively larger distances, due to the presence of the rotational coupling, the Businaro-Gallone barrier shifts towards higher mass asymmetry, thus favoring more mass equilibration towards symmetric configuration. The consequence of this is the occurrence of preequilibrium fission events irrespective of the entrance channel mass asymmetry values, at sub-barrier energies. Hence, even for systems like \( ^{11}\text{B}+^{235}\text{U} \) \( (\alpha>\alpha_{BG}) \), there will be preequilibrium fission components in addition to equilibrium fission at sub-barrier energies. However, at above barrier energies, there will be only equilibrium fission components for these systems as per the original model of Ref. [10], which does not take into account the orientation dependence. It is to be noted that this model has been tested for a large body of anisotropy data for actinide targets, and the model has good predictive power [38]. Following the formalism of Thomas et al. [38], the fission fragment anisotropy values were calculated for both the \( ^{11}\text{B}+^{235}\text{U} \) and \( ^{14}\text{N}+^{232}\text{Th} \) systems. Both the systems involve nonzero target and projectile ground state spins, which affect the \( K \) distribution at sub-barrier energies. The spin effect has also been incorporated while calculating the angular distribution by the above model. The results of the calculations are shown in Fig. 11. The above calculation explains the data satisfactorily. It may be mentioned that even though both quasifission and preequilibrium approaches account for the anisotropy data, the quasifission model is somewhat limited, as it requires knowledge of energy dependence of quasifission anisotropy.

VI. SUMMARY AND CONCLUSION

Fission fragment angular distributions for the two systems, \( ^{11}\text{B}+^{235}\text{U} \) and \( ^{14}\text{N}+^{232}\text{Th} \), having different entrance channel mass asymmetries and ground state target spins, but populating the same compound nucleus and at similar excitation energies, have been reported. In the analysis, bombarding energies have been separated into two parts, above barrier and near the barrier region. Incomplete momentum transfer events for \( ^{14}\text{N}+^{232}\text{Th} \) have been separated around a 90° center-of-mass angle. Contribution of the incomplete momentum transfer events is found to be 5–8% in the entire energy range of investigation. The measured anisotropy data for \( ^{11}\text{B}+^{235}\text{U} \) are found to be consistent with the SSPM prediction, whereas anisotropy data for \( ^{14}\text{N}+^{232}\text{Th} \) are found to be anomalous with respect to the SSPM. Both quasifission and preequilibrium fission models have been employed to explain the energy dependence of the anisotropy data successfully. The effects due to both target and projectile spins have been taken into account in the model calculation using the preequilibrium fission model. The overall agreement between the calculation and the experiment over the entire energy range can be considered satisfactory.

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