Two-quasiparticle isomer, $E1$ hindrances and residual interactions in $^{172}$Tm

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The structure of the neutron-rich nucleus $^{172}$Tm has been studied using incomplete fusion of $^7$Li on an $^{170}$Er target at 30 MeV. A 190-μs isomer at an excitation energy of 476 keV was identified using chopped beams and $\gamma$-ray spectroscopy. The isomer decays with very inhibited $E1$ transitions to the rotational bands based on the parallel and antiparallel couplings of the $ν5/2^- [512] \otimes π1/2^+ [411]$ configuration, the latter ($K'' = 2^+$) being the ground state. The isomeric state has been assigned $J^π = 6^+$, arising from the energetically favored (parallel) coupling of the $ν5/2^- [512] \otimes π7/2^- [523]$ configuration. The proton-neutron residual interaction was deduced for the configuration of the isomeric state and is found to agree with previous empirical studies.

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I. INTRODUCTION

The deformed nuclei in the neutron-rich rare-earth region have neutron and proton Fermi surfaces that lie close in energy to high-$Ω$ orbitals. This leads to many low-lying, multi-quasiparticle intrinsic states with large $K$, where $K = \sum_\Omega Ω_i$, the projection of the total intrinsic angular momentum on the symmetry axis. Frequently, these intrinsic states display long lifetimes because of large decay hindrances, $F$, defined as the ratio of the partial $γ$-ray mean-lives to the Weisskopf estimate. These large hindrances arise when decay from the high-$K$ states to lower-$K$ states requires a change in $K$ greater than the transition multipolarity, $λ$. The shortfall is defined as the forbiddenness, $ν = ΔK - λ$, and for a forbidden transition ($ν \geq 1$), the reduced hindrance is given by $f_ν = F^{1/ν}$.

Neutron-rich nuclei in this region can be difficult to study because usually they cannot be populated in traditional fusion-evaporation reactions, the benchmark technique for high-spin spectroscopy. As a result, there are gaps in the experimental data available. One such area is for nuclei with $Z \leq 70$, where very little information exists for proton and neutron intrinsic states beyond neutron number $N = 102$. For example, in the holmium isotopes ($Z = 67$), spectroscopic data is available only up to $A = 169$. In the erbium isotopes ($Z = 68$), $^{171}$Er, $^{172}$Er, and $^{174}$Er have been studied [1,2] but $^{173}$Er is unknown. In the thulium isotopes ($Z = 69$), the data for $A = 172$ and above come primarily from $β$-decay studies, where the states that can be populated are constrained by the spin and parity of the $β$-decaying states in the parent nucleus. Such is the case for the subject of the present article, $^{172}$Tm, which has previously been studied only via $β$ decay of the $0^+$ ground state in $^{172}$Er [3], with the resultant decay scheme being limited to states with $J \leq 3$.

In recent years, breakup reactions have been utilized to simulate low-mass radioactive ion beams, enabling the population of nuclei a few neutrons beyond the limits of fusion-evaporation reactions [4]. In the current work, we have utilized this technique to populate higher spins in $^{172}$Tm with a focus on isomeric states. The present article will discuss a newly observed isomer and the low-lying structure in $^{172}$Tm, with specific focus on the decay properties of the isomer and the properties of newly observed rotational bands.

II. EXPERIMENTAL PROCEDURE

Measurements were performed using the 14UD Pelletron accelerator at the Australian National University. A $^7$Li beam was incident on a 97% enriched, 3 mg/cm$^2$ $^{170}$Er target. The most prolific residual nuclei in this reaction are $^{174}$Lu ($3n$ channel), $^{173}$Lu ($4n$), and $^{171}$Tm ($2n$) with $^{172}$Tm ($\alpha n$) being very weakly produced. A beam energy of 30 MeV (just above the nominal Coulomb barrier for $^7$Li+$^{170}$Er) was chosen to maximize the yield of $^{172}$Tm. The ($\alpha n$) channel populating $^{172}$Tm proceeds predominantly via $^7$Li breakup into $α+3^1$H, effectively simulating the reaction $^{170}$Er($^3$H,$n$)$^{172}$Tm and, in principle, populating states up to $\sim 106$.

Two measurements were performed, one with a beam-chopping condition of 300 μs/1800 μs on/off, applied to isolate long-lived isomorphic states, the other with 107 ns/1600 ns on/off, to study the prompt population and de-excitation from short-lived isomers. Events were collected with a ±856-ns overlap, $γ$-$γ$ trigger using the CAESAR array [5], consisting of nine HPGe detectors and two LEPS detectors. For the longer chopping regime, only out-of-beam data were collected and timing information for each event with respect to the beam was obtained from an ADC clock, allowing the measurement of isomer lifetimes in the range of $μ$s to ms.

As well as $γ$-$γ$ coincidence data, a singles excitation function was performed at beam energies between 30 and 46 MeV, in 4-MeV increments, to evaluate the relative yields and assist in isotopic assignments of $γ$ rays.

III. RESULTS

For the $γ$-ray coincidence analysis, out-of-beam $γ$-$γ$ matrices were produced by taking time cuts on the out-of-beam

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FIG. 1. Coincidence spectra for γ rays attributed to $^{172}\text{Tm}$, in the out-of-beam time region. Filled circles indicate known contaminant lines. (a) γ rays coincident with the 102-keV line; the inset shows the same gate but projecting the LEPS detectors. (b) Sum of the 104- and 187-keV coincidence spectra used for the intensity balance of the 102-keV line. (c) 35-keV gate showing the 3 rotational band and decay to the ground-state band; the inset shows a 240-keV gate, projecting coincidences in the LEPS detectors.

time regions of the 300-μs/1800-μs chopped data, with a ±170-ns overlap requirement between pairs of γ rays. Lifetime measurements were performed using a γ-γ-$t_b$ cube, where $t_b$ is the time of detection for the coincidence event with respect to the chopped beam from the ADC clock, with a ±170-ns relative time condition on the γ rays.

The RADWARE [6] and ROOT [7] software packages and the local program DCP were used in the analysis. The γ rays observed in the 300-μs/1800-μs out-of-beam γ-γ matrix were attributed to the decay of isomers with lifetimes longer than tens of μs and to states populated in β decay. The observed γ rays were assigned to specific nuclei through coincidences with characteristic x rays, yield information from the excitation function and coincidences with γ rays from known decay schemes. The most intense γ rays in the out-of-beam region originate from the de-excitation of the 7/2$^-$[523] isomer in $^{171}\text{Tm}$, populated in the β decay of $^{171}\text{Er}$ (94% branch) [8,9], which is presumably produced from neutron transfer on the $^{170}\text{Er}$ target.

Figures 1(a), 1(b), and 1(c) show out-of-beam coincidence spectra for various γ rays identified as belonging to a previously unknown structure and assigned to a thulium isotope from their coincidences with the characteristic $K_\alpha$ and $K_\beta$ x-ray doublets.

The relative yield of γ rays associated with this structure were compared with γ rays known to belong to $^{171}\text{Tm}$ [10] and $^{173}\text{Lu}$ [11] in the excitation function. Its relative yield is highest in the 30-MeV measurement, whereas the relative yields of structures belonging to $^{171}\text{Tm}$ and $^{173}\text{Lu}$ are lower at 30 MeV than at higher energies. Of the thulium isotopes, this behavior is consistent only with that expected for the yield in $^{172}\text{Tm}$, supporting the assignment.

The decay scheme shown in Fig. 2 was based initially on the coincidence information obtained from the out-of-beam data, taken in the long chopping region. Subsequent analysis of the in-beam data (with the short-chopping conditions) led to tentative extensions of the rotational bands as shown in the figure. The transitions and levels up to the 146- and 240-keV states are known from the literature [3,12], in which configuration assignments were also proposed for the 2$^-$ ground state and the 240 keV, 3$^-$ bandhead. The present results confirm these configuration assignments, as discussed in Sec. IV.

On the basis of the observed branches to 5$^-$ and 6$^-$ states, the spin and parity of the isomeric state is restricted to $J^\pi = 6^-$.

FIG. 2. The proposed decay scheme for $^{172}\text{Tm}$. Dashed levels and γ rays are tentative extensions deduced from the in-beam data. The 1$^+$ state at 610 keV is known from β-decay studies [3].
TABLE I. Branching ratios and \( |g_{K^-g_K}| \) values for the \( 2^- \) and \( 3^- \) rotational bands in \( ^{172}\text{Tm} \).

| \( J^\pi \) | \( E_j(\Delta I = 1) \) (keV) | \( E_j(\Delta I = 2) \) (keV) | \( \lambda^a \) | \( |g_{K^-g_K}|^b \) | exp |
|---|---|---|---|---|---|
| \( K^\pi = 2^- \) | | | | | |
| \( 6^- \) | 125 | 229 | 1.77(7) | 0.65(2) | |
| \( 5^- \) | 104 | 187 | 0.90(5) | 0.68(3) | |
| \( 4^- \) | 83 | 146 | 0.36(6) | 0.67(4) | |
| \( K^\pi = 3^- \) | | | | | |
| \( 5^- \) | 112 | 202 | 0.23(8) | 0.75(14) | |

\(^a\lambda = I_j(E2)/I_j(M1).\)

\(^b\) Taking \( Q_0 = 7.7 \text{ eb}.\)

or \( \gamma^\pm \). To constrain the assignment, the multipolarities of the de-exciting transitions were deduced by extracting total conversion coefficients, \( \alpha_T \), from intensity balances. For the 102-keV \( \gamma \)-ray, the intensities of the 102- and 125-keV \( \gamma \)-rays are compared in Fig. 1(b). As a rotational band member, the 125-keV \( \gamma \)-ray will be a mixed \( E2/M1 \) transition with a mixing ratio \( \delta \) that can be deduced [using Eq. (2)] from the branching ratios of competing cascade and crossover transitions in the band, shown in Table I. A similar procedure was undertaken for the 35-keV transition, by balancing the intensities of the 35- and 112-keV transitions in the 90-keV gated spectrum. For the 226-keV transition, the sum of 83- and 146-keV gates was used and \( I_{\text{tot}}(226) \) balanced against \( I_{\text{tot}}(104) \), after accounting for feeding from the \( 6^- \) state. For the 102-keV transition, a total conversion coefficient was obtained of \( \alpha_T(102) = 0.4(1) \), to be compared with predicted values [13] of 0.32(\(E1\)), 2.95(\(M1\)), 2.87(\(E2\)), and 27.2(\(M2\)).

The 35- and 102-keV transitions have clear \( E2 \) assignments. The \( M2 \) multipolarity can be excluded for the 226-keV transition, therefore it must also be \( E1 \). Because the 35-keV transition decays to a \( 5^- \) state, its \( E1 \) multipolarity constrains the isomer to \( J^\pi = 6^- \), from the possible \( J^\pi = 6^\pm \) or \( 7^\pm \) assignments.

The decay curve for the isomer is shown in Fig. 3. The time spectrum was produced from the \( \gamma \)-\( \gamma \)-\( E4 \) cube and shows the summed intensity as a function of time for the 83-, 125-, 187-, and 229-keV\( \gamma \)-rays, gated by the 102-keV transition.

IV. DISCUSSION

A. \( 2^- \) and \( 3^- \) band magnetic properties

For an odd-odd nucleus, low-lying intrinsic states are characterized by two-quasiparticle excitations of the unpaired neutron and proton. In such a configuration, the individual spin projections \( \Omega \), combine to give \( K = |\Omega_1 \pm \Omega_2| \), where (+) indicates a parallel and (−) an antiparallel coupling. The two intrinsic states that would otherwise be energy-degenerate are split by the proton-neutron residual interaction. The shifts for parallel and antiparallel couplings are equal in magnitude but opposite in sign, the energy-favored coupling being given by the Gallagher-Moszkowski rules [14].

In Ref. [3], Hansen et al. observed a state at 610 keV that was deduced to have \( J^\pi = 1^+ \) because of its population by a relatively fast \( \beta \) decay from the \( 0^+ \) ground state in \( ^{172}\text{Er} \) and its decay by a 610-keV \( E1 \) transition to the \( 2^- \) ground state. The state was assigned as the antiparallel coupling of the \( 5/2^- \) \([512] \otimes 7/2^+ \) [523] configuration. The 476-keV \( 6^+ \) isomer observed in the current work is suggested to be the parallel coupling of the same configuration. This is supported by the Gallagher-Moszkowski rules that predict that the \( 6^+ \) parallel coupling will be favored in energy.

The \( 2^- \) ground-state band in \( ^{172}\text{Tm} \) has been proposed by Hansen et al. to be the antiparallel coupling of the \( 5/2^- \) \([512] \otimes 1/2^+ \) [411] configuration, whereas Meyer and Lanier (private communication, see Ref. [12]) reported a tentative \( 3^- \) intrinsic state at 240 keV decaying via 240- and 177-keV \( \gamma \)-rays to the ground state and first excited states, respectively. This state was proposed to be the parallel coupling of the \( 5/2^- \) \([512] \otimes 1/2^+ \) [411] configuration. The first few states in both of these rotational bands are now identified in the present work, as shown in Fig. 2.

To examine the magnetic properties of the \( 2^- \) and \( 3^- \) bands, as well as to confirm the proposed configuration assignments, the magnitudes of \( g_{K^-g_K} \) were deduced from the relation

\[
\delta = 0.933 \times \frac{E_\gamma(I \rightarrow I - 1)}{\sqrt{I^2 - 1}} \frac{Q_0}{g_{K^-g_K}}. \tag{1}
\]

The \( E2/M1 \) mixing ratios, \( \delta \), in the \( K^\pi = 3^- \) and \( 2^- \) rotational bands were deduced from the observed crossover/cascade branching ratio, \( \lambda = I_j(E2)/I_j(M1) \), using the standard formula

\[
\frac{1}{\delta^2} = \frac{1}{\lambda} \left[ \frac{E_\gamma(I \rightarrow I - 2)}{E_\gamma(I \rightarrow I - 1)} \right]^2 \left[ \frac{I(K20)(I - 2K)}{I(K20)I - 1K} \right]^2 - 1. \tag{2}
\]

The branching ratios and deduced \( g_{K^-g_K} \) magnitudes are given in Table I. The values of \( |g_{K^-g_K}| \) can be compared with...
predictions obtained with the equation

\[ g_{K^*} = \frac{1}{K} \sum_i (\Omega_i g_{\Omega i}) - g_R, \tag{3} \]

using \( g_R = +0.33(4) \) [15] and empirical values for the \( g_2 \) of the individual Nilsson states: \( g_2(\pi 5/2^-[512]) = -0.42(3) \) (extracted from the \( E2/M1 \) branching ratios in \( ^{173}\text{Yb} \), Ref. [16]) and \( g_2(\pi 7/2^-[523]) = +1.25(8) \) (from new results in \( ^{173,175}\text{Tm} \) [17]). Values for \( g_2(\pi 1/2^+[411]) \) are observed to vary considerably in thulium isotopes, with those in the odd-even nuclei ranging from \(-1.5 \) to \(-2 \) (see, for example, Refs. [10,18]). Kern and Struble [15] observed that \( g_2(\pi 1/2^+[411]) \) for odd-odd thulium isotopes is significantly lower than that implied by the odd-A cases, with a value of \( g_2(\pi 1/2^+[411]) = -1.14(7) \) for \( ^{170}\text{Tm} \). In the present analyses, predicted values for \( g_{K^*}g_R \) are calculated using both the value given by Kern and Struble, and an average value of \( g_2(\pi 1/2^+[411]) = -1.75(25) \) from the odd-even measurements.

Table II compares the measured and predicted magnitudes of \( g_{K^*}g_R \) for the various configurations. The values for both parallel and antiparallel couplings of the \( \nu 5/2^-[512] \otimes \pi 1/2^+[411] \) are reproduced well with the odd-odd value of \( g_2(\pi 1/2^+[411]) = -1.14(7) \), while poorer agreement is seen using the value for odd-even nuclei of \( g_2(\pi 1/2^+[411]) = -1.75(25) \). This provides indirect support for Kern and Struble’s result that the value of \( g_2(\pi 1/2^+[411]) \) is lower in the odd-odd thulium nuclei.

### B. Transition strengths for decays from the 6^+ isomer

Table III gives measured transition strengths and hindrances for the transitions de-exciting the 6^+ isomer. Two values are listed for the reduced hindrances, the second (in italics) includes an additional factor of \( 10^4 \) in the expected hindrance to account for the fact that \( E1 \) transitions are already hindered with respect to the Weisskopf estimate. This renormalized value will be used in the subsequent discussion as indicated by italics.

The 35- and 226-keV transitions have very similar and large reduced hindrances, whereas that for the 102-keV transition is much smaller. The 102- and 226-keV transitions decay to the same band, with only the final spins being different. The expected \( B(E1; 102\text{-keV})/B(E1; 226\text{-keV}) \) ratio can be deduced from the \( K^- \)-forbidden modification of the Alaga rule given by Bohr and Mottelson [19] that yields a value of 0.93, hence the transitions are expected to have similar reduced hindrances. It is unclear why the experimentally observed reduced hindrances differ so significantly.

In the neighboring odd-mass thulium isotopes, the 7/2^-[523] single-proton state decays predominantly via \( E1 \) transitions to the 7/2^- and 5/2^+ members of the 1/2^+[411] band. These \( E1 \) transitions are analogous to the 102- and 226-keV transitions in \( ^{172}\text{Tm} \). In \( ^{173}\text{Tm} \) [10], the corresponding 296- and 308-keV \( E1 \) decays have hindrances of \( F = 1.1 \times 10^6 \) and \( F = 5.5 \times 10^5 \) and reduced hindrances of \( f_v = 330 \) and \( f_v = 235 \), respectively. In \( ^{173}\text{Tm} \) [20], the related 193- and 199-keV transitions have hindrances of \( F = 8.3 \times 10^5 \) and \( F = 6.5 \times 10^5 \) and reduced hindrances of \( f_v = 288 \) and \( f_v = 255 \), respectively. In both cases the transitions to the 7/2^- and 5/2^+ states have \( f_v \) values which are large, as in \( ^{172}\text{Tm} \), but are similar in magnitude, contrary to what is seen in the \( ^{172}\text{Tm} \) case.

In the odd-odd neighbor, \( ^{170}\text{Tm} \), Andreeff et al. [21], assigned a 3^+, 4.1-\( \mu \)s isomer to originate from the antiparallel coupling of the \( \nu 7/2^+[633] \otimes \pi 1/2^+[411] \) configuration. It decays by 68- and 144-keV \( E1 \) transitions with forbiddenness, \( v = 1 \), to the 1^- ground-state band, with hindrance values of \( F = 3.5 \times 10^7 \) and \( F = 8.5 \times 10^7 \), respectively, and, including the additional \( 10^4 \) factor, correcting reduced hindrances of \( f_v = 3500 \) and \( f_v = 8500 \). Despite the extra factor, the hindrances remain anomalously high. Andreeff et al. proposed an additional selection rule on forbiddenness.

### Table II. Values of \( g_{K^*}g_R \) for proposed configurations.

| \( K^* \) | Configuration \( \nu \pi \) | \( g_{K^*}g_R \) predicted | \( g_{K^*}g_R \) predicted | \( |g_{K^*}g_R| \) measured |
|---|---|---|---|---|
| 2^- | 5/2^-[512] \otimes -1/2^+[411] | -0.42(25) | -0.57(9) | 0.67(3) |
| 3^- | 5/2^-[512] \otimes 1/2^+[411] | -0.97(25) | -0.87(9) | 0.75(14) |
| 6^+ | 5/2^-[512] \otimes 7/2^-[523] | +0.22(10) |   |   |
| 1^+ | 5/2^-[512] \otimes -7/2^-[523] | +5.1(1) |   |   |

*Taking \( g_2(\pi 1/2^+[411]) = -1.75(25) \).

*Taking \( g_2(\pi 1/2^+[411]) = -1.14(7) \).

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<th>( I_v ) relative</th>
<th>( M_\lambda )</th>
<th>( \alpha_\gamma )</th>
<th>( B(E1) ) (e^2 fm^2)</th>
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*Values in italics include an additional factor of \( 10^4 \) in the expected single-particle hindrance for \( E1 \) transitions.
specific to a configuration change between two-quasiparticle states, where one quasiparticle undergoes a spin-flip ($\Omega$ is in opposite directions in initial and final states), whereas the other quasiparticle remains the same in both configurations. Andreeff’s application of this rule results in an effective forbiddenness, $\nu' = 3$, which brings the reduced hindrances down to values of $f_{\nu'}(68-\text{keV}) = 15$ and $f_{\nu'}(144 \text{ keV}) = 20$, which are very small, but note that these again include an additional (somewhat arbitrary) $10^7$ factor. These $f_{\nu'}$ values are, however, closer in magnitude to each other than those observed for the 102- and 226-keV transitions in $^{172}$Tm.

### C. Residual interactions and multi-quasiparticle calculations

As discussed in Sec. IV A, residual two-body interactions lead to equal and opposite energy shifts in parallel and antiparallel couplings of a given two-quasiparticle configuration. The residual interactions, $V_{\text{res}}$, can be extracted from the experimentally observed energies, $E_{\pm}$ of the two couplings using

$$E_{\pm} = E_{2qp} \pm V_{\text{res}} + E_{\text{rot}},$$

where $E_{2qp}$ is the unperturbed energy given by the coupling of the two quasiparticles, $V_{\text{res}}$ is the residual interaction (which will be positive or negative depending on the nucleon coupling), and $E_{\text{rot}}$ is a Coriolis correction, which can, however, be neglected because Coriolis effects are expected to be small in the configurations discussed. $E_{\text{rot}}$ is a rotational energy contribution given by

$$E_{\text{rot}} = \frac{\hbar^2}{2I} \times K = \frac{E_{\gamma}(I \rightarrow I - 1)}{2I} \times K,$$

where the moment of inertia for rotation of a given two-quasiparticle state $I$ is estimated using the energy of the first in-band cascade transition, $E_{\gamma}(I \rightarrow I - 1)$. An average value of $\hbar^2/2I = 10.8 \text{ keV}$ is calculated for the $2^-$ and $3^-$ bands.

In the case of the $2^-$ and $3^-$ states, a residual interaction of $\pm 114 \text{ keV}$ is deduced, very close to the empirical value of $116.5 \text{ keV}$ quoted by Kondev [22]. For the $6^+$ and $1^+$ states, the residual interaction is found to be $\pm 94 \text{ keV}$, in good agreement with the residual interaction derived from the $1^+$ and $6^+$ energy difference in $^{166}$Ho of $131 \text{ keV}$ (Headly et al., [23]), leading to $V_{\text{res}} = 92.5 \text{ keV}$ and, in $^{168}$Ho, $134 \text{ keV}$ [24], giving $V_{\text{res}} = 94 \text{ keV}$, Kondev [22] and Jain et al. [25] both quote the same residual interaction, purportedly taken from the $^{166}$Ho values reported by Headly et al.; however, their value of $V_{\text{res}} = 159 \text{ keV}$ is inconsistent with the experimental energy difference supposedly used.

A multi-quasiparticle calculation was performed similar to that outlined in Refs. [22,26]. Values for the deformation were taken to be $\epsilon_2 = 0.264$, $\epsilon_4 = 0.043$, from the average deformations of the neighboring even-even nuclei $^{170}$Er, $^{172}$Yb, and $^{174}$Yb [27]. Neutron and proton pairing strengths of $G_{\pi} = 17 \text{ MeV/nucleon}$ and $G_{\nu} = 23 \text{ MeV/nucleon}$ were used. The Nilsson energies of neutron and proton states close to the Fermi surface were adjusted to approximately match the quasiparticle energies of intrinsic states in the neighboring odd-mass nuclei.

$^{171}$Tm [10], $^{171}$Er [9], $^{173}$Tm [20], and $^{173}$Yb [16]. The calculated energies were corrected for residual interactions and rotational energy, and in the case of the $0^-$ state from the $1/2^- [521] \otimes \pi 1/2^+$ configuration, a Newby shift correction of $-29 \text{ keV}$ [25] was also included. Figure 4 compares the experimentally observed states (left) with the calculation (right). Only states below $700 \text{ keV}$ are shown.

The calculation reproduces all of the experimentally observed states reasonably well, with the exception of the $0^-$ and $1^-$ states observed in Ref. [3], arising from the $1/2^-[521] \otimes \pi 1/2^+$ configuration, which differ in both energy splitting (68 keV in experiment compared to 150 keV in the calculation) and excitation energy. The energy of this configuration is very sensitive to the energy used for the $1/2^- [521]$ quasineutron, for which the exact value is uncertain. For example, this state is observed in the odd-mass neighbours at $199 \text{ keV}$ ($^{171}$Er [9]) and $399 \text{ keV}$ ($^{173}$Yb [16]). The Nilsson energy for the $1/2^- [521]$ state used in the present calculation was chosen so that the calculated quasiparticle energy reproduced the mean of the values in the odd-mass neighbors.

The calculation supports an assignment of the $2/5^- [512] \otimes \pi 7/2^-$ configuration to the $6^+$ level. No other states with $K > 4$ are predicted to occur below $700 \text{ keV}$, with the limited available decay paths leading to a long lifetime for the $6^+$ intrinsic state.

### V. CONCLUSIONS

A decay scheme for the neutron-rich nucleus $^{172}$Tm has been obtained from $\gamma$-ray spectroscopic studies using incomplete fusion of $30-\text{MeV} \ ^{7}\text{Li}$ on a $^{170}$Er target. New results include the identification of a two-quasiparticle, $K^\pi = 6^+$ isomer interpreted as the parallel coupling of the $5/2^- [512] \otimes \pi 7/2^-$ configuration. The isomer is found to decay via three $E1$ transitions to rotational bands built on the $2^-$ and $3^-$ intrinsic states from the $5/2^- [512] \otimes \pi 1/2^+$ configuration. Support for the previous configuration assignments to the $2^-$ and $3^-$ states in $^{172}$Tm is obtained from the branching ratios and magnetic properties within the bands. The difference in the experimental bandhead energies of the $6^+$ isomer and the previously assigned $1^+$ state from the antiparallel coupling results in a new empirical value of $\pm 94 \text{ keV}$ for the
residual interaction strength in the $\nu 5/2^-[512] \otimes \pi 7/2^-[523]$ configuration, which is consistent with values measured for this configuration in the odd-odd holmium isotopes.

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