$^{15}N(p, \alpha_0)^{12}C$ S factor

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Experimental values of the astrophysical S factor for the $^{15}N(p, \alpha_0)^{12}C$ reaction are available both from direct measurements and from the Trojan horse method. We here use R-matrix formulas to fit these values and to extrapolate to zero energy to obtain values of $S(0)$.

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I. INTRODUCTION

La Cognata *et al.* [1] have recently measured the astrophysical $S(E)$ factor for the $^{15}N(p, \alpha_0)^{12}C$ reaction using the Trojan horse method (THM) and extrapolated to zero energy using the half-off-energy-shell (HOES) R-matrix approach. They obtain $S(0) = 63$–68 MeV b, with uncertainties of order 10 MeV b, in agreement with previous values obtained from direct measurements of the $^{15}N(p, \alpha_0)^{12}C$ cross section [2–4].

Schardt, Fowler, and Lauritsen [2] fitted their data for proton energies $E_p = 0.2$–0.4 MeV using a one-level approximation representing the 12.44 MeV 1^- level of ^{16}O. They obtained $S(0) = 103$ MeV b, but noted that destructive interference with the contribution from the 13.09 MeV 1^- level could reduce this by a factor of 2. Hebbard [5] fitted the data of Schardt, Fowler, and Lauritsen [2] using an R-matrix two-level approximation, obtaining $S(E_p = 25\ \text{keV}) = 72$ MeV b. From this, Zyskind and Parker [3] deduced $S(0) \approx 64$ MeV b.

Zyskind and Parker [3] fitted their own data, normalized to the Schardt, Fowler, and Lauritsen 12.44 MeV peak [2], for $E_p = 93$–418 keV using a two-level approximation of the form given by Rolfs and Rodney [6] and obtained $S(0) = 78(6)$ MeV b. The same form was used by Redder *et al.* [4] to fit their data for $E_p = 78$–810 keV, giving $S(0) = 65(4)$ MeV b. This last value was adopted by Angulo *et al.* [7] in the NACRE compilation.

La Cognata *et al.* [1] fit their THM S factor for $E_{\text{c.m.}} = 19$–516 keV in two ways, one using the HOES R-matrix approach and the other assuming a functional form of a second-order polynomial plus a Breit-Wigner function, as given in their Eq. (32). There seems to be little justification for such a form, which has energy-independent W and Γ, resulting in the polynomial being negative for all $E < E_R$; nevertheless the data extend to such low energies that the uncertainty in the extrapolated value $S(0) = 62$ MeV b due to the extrapolation should be small. La Cognata *et al.* [1] also use the two-level HOES R-matrix expression given in their Eq. (33). In principle, the S factor measured in the THM can be different from that obtained in direct measurements. This is shown, for example, by Eqs. (33) and (34) in Ref. [1] [Eq. (34), which is applicable to direct measurements, is essentially the Rolfs and Rodney [6] formula]. The S factors are equal only if M_{21}^0, the ratio of the transfer reaction amplitudes to the two levels, is equal to $\gamma_{Ax(21)}$, the ratio of their reduced-width amplitudes. In the present case, this seems to be satisfied, as La Cognata *et al.* [1] find $M_{21}^0 \approx 1.13$ and $\gamma_{Ax(21)} = 1.1 \pm 0.1$.

This agreement in the values of M_{21}^0 and $\gamma_{Ax(21)}$ appears to be accidental. For example, Rolfs and Rodney [6] give $\theta^2 = 0.20$ and 0.14 for the 12.44 and 13.09 MeV levels, respectively, leading to $\gamma_{Ax(21)} = 0.84$. Also the departure of M_{21}^0 from unity, as calculated by La Cognata *et al.* [1], is due only to the different energies of the two levels, while the value of $\gamma_{Ax(21)}$ depends on the structure of the levels (including isospin mixing).

Rolfs and Rodney [6] do not describe their two-level expression as an R-matrix formula. Why do La Cognata *et al.* [1] describe theirs as "R-matrix"? The form of Eq. (33) in La Cognata *et al.* comes from their Eq. (9), which is taken from Eq. (13) in Ref. [8] (this is essentially La Cognata *et al.*'s Ref. [53]). Reference [8] says that their Eq. (13) is "for a simple case when the distances between the two resonances are significantly larger than their total widths" and justifies it by analogy with an R-matrix formula given by Lane and Thomas [9] in their Eq. (XII.5.15). Lane and Thomas give this formula with an apparently stronger condition on its validity—$\Gamma/D \ll 1$. La Cognata *et al.*, however, do not mention such restrictions on the validity of their Eqs. (9) and (33); they merely say that their Eq. (9) "is like the two-level, two-channel R-matrix amplitude when the distance between the interfering resonances is significantly larger than the widths of the resonances." The importance of interference between the 12.44 and 13.09 MeV levels has been recognized since early measurements [2], so it seems desirable not to make approximations that assume the interference is small.

There is also a significant difference between the Lane and Thomas [9] R-matrix formula (XII.5.15) and the Eqs. (13) of Ref. [8] and Eqs. (33) and (34) of Ref. [1]. The R-matrix formula contains level-shift terms $\Delta_\lambda = \sum_c \gamma_{\lambda c}^2 (B_c - S_c(E))$. They have been omitted in Refs. [1] and [8], without comment in the two-level approximation. Reference [8] justifies omission of Δ_λ in the one-level approximation by saying that the formula involves "observable" widths $\Gamma_c(E)$, related to "formal" widths $\tilde{\Gamma}_c(E)$ by $\Gamma_c(E) = \tilde{\Gamma}_c(E)/(1 - \mathrm{d}\Delta/\mathrm{d}E)_{E_r}$, with E_r being the resonance energy. This assumes that $B_c = S_c(E_r)$. This justification cannot be used in the two-level approximation, as R-matrix theory [9] requires B_c to be the same for all levels.

There seems to be little reason for Ref. [1] describing their Eqs. (33) and (34) as "R-matrix", while they describe the Rolfs and Rodney [6] two-level expression as "Breit-Wigner." There have been two applications of genuine R-matrix formulas to the direct $^{15}N(p, \alpha_0)^{12}C$ reaction, by Hebbard [5] (details of calculation not given) and by Bray *et al.* [10] (for higher energies). *R*-matrix formulas that could be used for the $^{15}N(p, \alpha_0)^{12}C$ reaction for either the direct or THM measurements have been given in Ref. [11] [Eq. (1) for the direct case and Eq. (4) for the THM case]. These equations are not restricted to well-separated levels and include the level shift through the quantity L_c^0. We now use these equations, with $N = 2, l = 0, s = 1$, to fit the direct data [3,4] and the THM data [1], which are all tabulated; the S factor in Ref. [2] is given only graphically. Equation (4) in Ref. [11] can be used for the $^{15}N(d, \alpha^{12}C)n$ reaction and must be multiplied by a $^{15}N + p$ penetration factor [as well as the usual $E \exp(2\pi\eta)$] before comparison with the THM S factor given in Ref. [1]. We neglect any effects of electron screening in the direct measurements, as they are expected to be small [1].

II. *R*-MATRIX FITS

Because of the small partial widths of the two 1^- levels for the $^{12}C(4.44) + \alpha$ and the γ channels [12], we neglect all contributions from these channels. We therefore use the two-level, two-channel approximation of the formulas in Ref. [11]. Then, for given values of the channel radii $a_c (c = p, \alpha)$ and boundary condition parameters B_c, there are six adjustable parameters in the fit to the direct data—the eigenenergies $E_\lambda (\lambda = 1, 2)$ and the reduced-width amplitudes $\gamma_{\lambda c}$. For fits to the THM data, there are in addition two feeding amplitudes $G_{\lambda x}^{1/2} = g_\lambda$, say (taken as energy independent).

Because the data do not extend to the energy of the upper level, we have to obtain the value of E_2, and perhaps of the γ_{2c}, from other sources. Most work, including Refs. [1,3,4,12], has used the resonance energy $E_{2r} = 0.9624$ MeV, corresponding to an excitation energy of 13.090 MeV. For convenience, we take $B_c = S_c(E_{2r})$, so that $E_2 = E_{2r}$. There is considerable uncertainty in the values of the partial widths Γ_{2p} and $\Gamma_{2\alpha}$, and the total width $\Gamma_2 = \Gamma_{2p} + \Gamma_{2\alpha}$. The latest $A = 16$ compilation [12] gives in Table 16.15 (from $^{12}C + \alpha$ reactions) $\Gamma_{2p} = 100$ keV, $\Gamma_{2\alpha} = 45(18)$ keV, but $\Gamma_2 = 130(5)$ keV. This value for $\Gamma_{2\alpha}$ comes from Ref. [13], which says that the values of Hebbard [5] are probably more accurate. The compilation gives Hebbard's values in Table 16.22 (from $^{15}N + p$ reactions): $\Gamma_{2p} = 100$ keV, $\Gamma_{2\alpha} = 40$ keV, and Γ_2 (laboratory) = 140(10) keV. These are all laboratory values [10], and they are all formal widths [14]; observed widths in the c.m. system would be about 80% of these. Zyskind and Parker [3] and Redder *et al.* [4] both used $\Gamma_{2p} = 100$ keV, $\Gamma_{2\alpha} = 30$ keV, and $\Gamma_2 = 130$ keV. La Cognata *et al.* [1] give $\Gamma_{2p} = 95.31$ keV and $\Gamma_{2\alpha} = 45$ keV; they say that these are obtained by fitting the direct measurements in Refs. [2–4], but only Ref. [2] covers the region of the upper 1^- level and Hebbard [5] included the data of Ref. [2] in his fits. Bray *et al.* [10] found $\Gamma_{2p}^0 = 111$ keV and $\Gamma_{2\alpha}^0 = 26$ keV. To cover (roughly) this range of values, we do fits with three sets of values of $\Gamma_{2p}, \Gamma_{2\alpha}$ (in keV): A 100, 30; B 100, 45; C 95, 45. In view of the evidence above, set A is preferred. To obtain values of γ_{2c}, we interpret these experimental widths as observed widths, given by

$$\Gamma_{2c}^0 = 2\gamma_{2c}^2 P_c(E_2) \Bigg/ \left[1 + \sum_{c'=p,\alpha} \gamma_{2c'}^2 (\mathrm{d}S_{c'}/\mathrm{d}E)_{E_2}\right], \quad (1)$$

provided $B_c = S_c(E_2)$.

From Ref. [11], we fit the S factors obtained in the direct measurements [3,4] with the formulas

$$S(E) = E\, e^{2\pi\eta} \left(3\pi/k_p^2\right) P_p(E) P_\alpha(E) \times \left| \sum_{\lambda,\mu=1}^{2} \gamma_{\lambda p}\gamma_{\mu\alpha} A_{\lambda\mu}(E) \right|^2, \quad (2)$$

$$(\mathbf{A}(E)^{-1})_{\lambda\mu} = (E_\lambda - E)\delta_{\lambda\mu} - \sum_{c=p,\alpha} \gamma_{\lambda c}\gamma_{\mu c}[S_c(E) - B_c + i\, P_c(E)]. \quad (3)$$

We take the uncertainty in the Zyskind and Parker [3]S factor as ±5% for $E_p \geqslant 213$ keV, the same as the uncertainty in the Redder *et al.* [4] data for $E_p \geqslant 107.4$ keV. Initially we use the conventional value $a_p = 1.45(15^{1/3} + 1)$ fm = 5.03 fm and $a_\alpha = 6.5$ fm [15]. Results of fits to the direct data [3,4] are given in Table I. For each of the cases A, B, and C, only the parameters of level 1 are adjusted. In row D, the reduced-width amplitudes for level 2 are also allowed to vary. The fits to the Zyskind and Parker data [3] have values of the reduced $\chi^2 (\chi_\nu^2 = \chi^2$/degree of freedom) much smaller than those of the fits to the full Redder *et al.* data [4]. When the reduced widths for level 2 are also allowed to vary (rows D), the fit to the Zyskind and Parker data is not improved, while for the Redder *et al.* data χ_ν^2 is reduced appreciably but the fit is unreasonable because it gives $\Gamma_{2p}^0 = 25$ keV and $\Gamma_{2\alpha}^0 = 268$ keV. It seems that the Zyskind and Parker data are consistent with the measured properties of the upper level, while the Redder *et al.* data would favor a larger width. For a better comparison with the Zyskind and Parker fits, Table I also gives fits to the Redder *et al.* data for $E_p \leqslant 418$ keV. Again fit D is unreasonable, while fits A–C are poorer than the Zyskind and Parker fits. Although the two sets of data agree well over the peak region, the Redder *et al.* values of $S(E)$ lie below the Zyskind and Parker values for $E_p \lesssim 200$ keV, and about half of χ^2 for the Redder *et al.* fits comes from this region.

For the Zyskind and Parker fits, our values of $S(0) \approx 80$ MeV b agree with theirs; for the Redder *et al.* fits, our values range from 74 to 82 MeV b (omitting the R71 A fit, for which χ_ν^2 is very high), which is considerably above their value of 65(4) MeV b.

The parameter values in Table I are for $B_c = S_c(E_2)$, so that E_2 is the resonance energy of the upper level, and its observed widths are given directly by Eq. (1). Exactly the same fits may be obtained for any choice of B_c, say B_c', provided the values of E_λ and $\gamma_{\lambda c}$ are adjusted suitably [16], giving the corresponding primed quantities. For $B_c' = S_c(E_1')$, then E_1' is the resonance energy of the lower level, and its observed widths are given by an equation similar to Eq. (1), but involving primed quantities. Table II gives the values of these quantities. The values of the energy and widths agree with those given in the compilation [12] from $^{12}C + \alpha$ reactions:

TABLE I. Fits to $^{15}N(p,\alpha_0)^{12}C$ direct data [3,4], using R-matrix two-level, two-channel formulas with $B_c = S_c(E_2)$ [$a_p = 5.03$ fm, $a_\alpha = 6.5$ fm].

Case		E_1 (MeV)	γ_{1p} (MeV$^{1/2}$)	$\gamma_{1\alpha}$ (MeV$^{1/2}$)	E_2 (MeV)	γ_{2p} (MeV$^{1/2}$)	$\gamma_{2\alpha}$ (MeV$^{1/2}$)	χ^2_ν	$S(0)$ (MeV b)
ZP21[a]	A	0.156	0.639	−0.112	0.9624[d]	0.522[d]	0.0635[d]	0.115	82
	B	0.153	0.641	−0.110	0.9624[d]	0.522[d]	0.0778[d]	0.111	77
	C	0.155	0.634	−0.110	0.9624[d]	0.506[d]	0.0775[d]	0.106	78
	D	0.169	0.583	−0.111	0.9624[d]	0.374	0.0903	0.114	79
R71[b]	A	0.156	0.650	−0.118	0.9624[d]	0.522[d]	0.0635[d]	5.04	92
	B	0.152	0.645	−0.112	0.9624[d]	0.522[d]	0.0778[d]	1.30	80
	C	0.154	0.639	−0.112	0.9624[d]	0.506[d]	0.0775[d]	1.58	82
	D	0.170	0.543	−0.108	0.9624[d]	0.247	0.180	0.59	69
R32[c]	A	0.156	0.634	−0.109	0.9624[d]	0.522[d]	0.0635[d]	1.84	78
	B	0.153	0.636	−0.108	0.9624[d]	0.522[d]	0.0778[d]	1.39	74
	C	0.156	0.630	−0.108	0.9624[d]	0.506[d]	0.0775[d]	1.44	75
	D	0.076	0.424	−0.263	0.9624[d]	0.385	0.666	0.90	56

[a]Zyskind and Parker [3], $E_p = 93.1$–418 keV, 21 data points.
[b]Redder *et al*. [4], $E_p = 77.6$–810 keV, 71 data points.
[c]Redder *et al*. [4], $E_p = 77.6$–418 keV, 32 data points.
[d]Fixed value.

$E_1 = 0.314(4)$ MeV, $\Gamma_{1p} = 1.1$ keV, $\Gamma_{1\alpha} = 92(8)$ keV, and $\Gamma_1 = 99(7)$ keV.

We now use the R-matrix formula Eq. (4) in Ref. [11] to fit the THM values of $S(E)$ given in Table III of La Cognata *et al*. [1]. To obtain their values of S, La Cognata *et al*. multiplied their measured cross section by a $^{15}N + p$ penetration factor, calculated with "a channel radius given by the sum of the radii of the two interacting nuclei." Apparently [17] this is $a_p = 1.40(15^{1/3} + 1)$ fm $= 4.85$ fm. In our fits, we are using $a_p = 5.03$ fm. The R-matrix formula is of the form of Eq. (2), but with $\gamma_{\lambda p}$ replaced by a feeding amplitude g_λ. Only the g_λ are now adjusted; values of the level energies and reduced-width amplitudes are taken from fits to the direct data [3,4] as given in Table I. Results are given in Table III. For our preferred fits A, g_2/g'_1 (which is equivalent to M^0_{21} in Ref. [1]) is 1.93–1.97, while γ_{2p}/γ'_{1p} (equivalent to $\gamma_{Ax(21)}$) is 0.98–1.00.

Because g_2/g'_1 is quite different from γ_{2p}/γ'_{1p}, the THM is measuring values of a quantity $S^{TH}(E)$, which is different from the $S(E)$ determined in direct measurements. Consequently the extrapolated values $S^{TH}(0)$ in Table III are not values of $S(0)$. For example, for the parameters in the first row ZP21 A of Table III, the value of $S(0)$ is 82 MeV b (from Table I).

Similar fits have also been made with two other sets of values of the channel radii: $a_p = 5.03$ fm, $a_\alpha = 5.5$ fm, and $a_p = 4.5$ fm, $a_\alpha = 6.5$ fm. In both cases χ^2_ν is little changed for the Zyskind and Parker fits, and is somewhat smaller for the Redder *et al*. and THM fits, while in all cases $S(0)$ and $S^{TH}(0)$ are changed by less than 1 MeV b.

Isospin mixing in the 12.44 and 13.09 MeV 1^- levels has been discussed previously [4–6]. In a two-state isospin-mixing model, the levels 1 and 2 may be written

$$|1\rangle = \alpha|T=0\rangle + \beta|T=1\rangle,$$
$$|2\rangle = \beta\,|T=0\rangle - \alpha|T=1\rangle, \quad (\alpha^2+\beta^2=1). \tag{4}$$

TABLE II. Fits to $^{15}N(p,\alpha_0)^{12}C$ direct data as in Table I, with $B_c = B'_c = S_c(E'_1)$.

Case		E'_1 (MeV)	γ'_{1p} (MeV$^{1/2}$)	$\gamma'_{1\alpha}$ (MeV$^{1/2}$)	Γ^0_{1p} (keV)	$\Gamma^0_{1\alpha}$ (keV)	Γ^0_1 (keV)
ZP21	A	0.314	0.526	−0.122	1.1	96	97
	B	0.313	0.527	−0.123	1.1	97	98
	C	0.313	0.527	−0.122	1.1	97	98
	D	0.312	0.526	−0.122	1.1	96	98
R71	A	0.318	0.534	−0.128	1.1	106	107
	B	0.314	0.531	−0.125	1.1	100	101
	C	0.314	0.531	−0.125	1.1	101	102
	D	0.303	0.519	−0.124	1.1	99	100
R32	A	0.312	0.522	−0.120	1.1	92	94
	B	0.311	0.524	−0.121	1.1	94	95
	C	0.311	0.523	−0.121	1.1	94	95
	D	0.172	0.389	−0.320	0.01	698	698

TABLE III. Fits to $^{15}N(p,\alpha_0)^{12}C$ THM S factor of La Cognata *et al.* [1], using R-matrix parameter values from Table I.

Case		g_1	g_2	χ_ν^2	$S^{TH}(0)$ (MeV b)	g_1'	g_2/g_1'	γ_{2p}/γ_{1p}'
ZP21	A	0.774	1.073	0.994	60	0.551	1.95	0.99
	B	0.738	0.895	0.862	60	0.550	1.63	0.99
	C	0.730	0.887	0.883	60	0.550	1.61	0.96
	D	0.652	0.696	0.945	60	0.550	1.27	0.70
R71	A	0.766	1.051	0.818	66	0.544	1.93	0.98
	B	0.737	0.891	0.780	62	0.548	1.63	0.98
	C	0.729	0.882	0.773	62	0.548	1.61	0.95
	D	0.574	0.357	0.827	57	0.540	0.66	0.48
R32	A	0.775	1.085	1.111	58	0.552	1.97	1.00
	B	0.740	0.908	0.954	58	0.551	1.65	1.00
	C	0.732	0.900	0.967	58	0.551	1.63	0.97
	D	0.437	0.454	1.705	48	0.395	1.15	0.99

Then $\gamma_{1\alpha}' = \alpha\gamma_{T=0,\alpha}$ and $\gamma_{2\alpha} = \beta\gamma_{T=0,\alpha}$, as given before, but also $\gamma_{1p}' = \alpha\gamma_{T=0,p} + \beta\gamma_{T=1,p}$ and $\gamma_{2p} = \beta\gamma_{T=0,p} - \alpha\gamma_{T=1,p}$. From the values in Tables I and II for the ZP21 A case, we find $\alpha = 0.887$, $\beta = -0.462$ and $\gamma_{T=0,p} = 0.226$ MeV$^{1/2}$, $\gamma_{T=1,p} = -0.706$ MeV$^{1/2}$. These values suggest that the $T = 0$ and $T = 1$ basic states have quite different structures, so that the near equality of γ_{1p}' and γ_{2p} appears to be accidental.

III. ROLFS AND RODNEY TYPE FITS

Table IV gives the results of fits to the same data as in Table I, but using the Rolfs and Rodney [6] expression instead of the R-matrix formula. In this case, widths are given by $\Gamma_{\lambda c} = 2\,\gamma_{\lambda c}^2 P_c(E_\lambda)$. We use the same penetration factors as before; the approximations given by Rolfs and Rodney lead to penetration factors that vary more rapidly with energy for both proton and α channels. Zyskind and Parker [3] and Redder *et al.* [4] do not say how they calculated penetration factors. Compared with the R-matrix fits in Table I, the Rolfs and Rodney expression gives similar fits to those of the Zyskind and Parker data, and generally better fits to the Redder *et al.* data, but the χ_ν^2 values are still much smaller for the Zyskind and Parker fits than for the Redder *et al.* fits. The parameter values in Table IV for the lower level are all much the same, and all give $\Gamma_{1p} = 1.1$ keV and $\Gamma_{1\alpha} = 95$–102 keV. For the upper level, the fits D all give unacceptable values; e.g., for R32, $\Gamma_{2p} = 317$ keV and $\Gamma_{2\alpha} = 23$ keV. The values of $S(0)$ in Table IV are generally smaller than the corresponding values in Table I; however, the values of $S(0)$ from our fits to the Redder *et al.* data are still appreciably larger than the Redder *et al.* value of 65(4) MeV b.

For a comparison with the results of La Cognata *et al.* [1], we give in Table V the values from fits to their data when a Rolfs and Rodney type expression (with g_λ replacing $\gamma_{\lambda p}$) is used instead of the R-matrix formula. For the preferred case A, $g_2/g_1 = 1.81$–1.84 while $\gamma_{2p}/\gamma_{1p} = 0.99$–1.01. For case C, for which the assumed values of Γ_{2p} and $\Gamma_{2\alpha}$ are close to those used by La Cognata *et al.*, we find $g_2/g_1 = 1.52$–1.54 and $\gamma_{2p}/\gamma_{1p} = 0.97$–0.98. These may be compared with the La Cognata *et al.* values: $M_{21}^0 \approx 1.13$ and $\gamma_{Ax(21)} = 1.1 \pm 0.1$.

TABLE IV. Fits to $^{15}N(p,\alpha_0)^{12}C$ direct data [3,4], using Rolfs and Rodney [6] two-level expression.

Case		E_1 (MeV)	γ_{1p} (MeV$^{1/2}$)	$\gamma_{1\alpha}$ (MeV$^{1/2}$)	E_2 (MeV)	γ_{2p} (MeV$^{1/2}$)	$\gamma_{2\alpha}$ (MeV$^{1/2}$)	χ_ν^2	$S(0)$ (MeV b)
ZP21	A	0.315	0.484	−0.112	0.9624[a]	0.485[a]	0.0590[a]	0.105	78
	B	0.314	0.486	−0.113	0.9624[a]	0.485[a]	0.0723[a]	0.170	74
	C	0.314	0.486	−0.113	0.9624[a]	0.473[a]	0.0723[a]	0.155	74
	D	0.315	0.484	−0.112	0.9624[a]	1.246	0.0221	0.116	79
R71	A	0.318	0.490	−0.117	0.9624[a]	0.485[a]	0.0590[a]	3.66	86
	B	0.314	0.487	−0.114	0.9624[a]	0.485[a]	0.0723[a]	0.772	74
	C	0.314	0.488	−0.114	0.9624[a]	0.473[a]	0.0723[a]	0.926	76
	D	0.312	0.486	−0.113	0.9624[a]	0.873	0.0438	0.604	70
R32	A	0.313	0.482	−0.111	0.9624[a]	0.485[a]	0.0590[a]	1.49	75
	B	0.312	0.484	−0.112	0.9624[a]	0.485[a]	0.0723[a]	1.16	71
	C	0.312	0.484	−0.112	0.9624[a]	0.473[a]	0.0723[a]	1.19	72
	D	0.310	0.486	−0.114	0.9624[a]	0.863	0.0522	1.04	65

[a]Fixed value.

TABLE V. Fits to $^{15}N(p, \alpha_0)^{12}C$ THM S factor of La Cognata *et al.* [1], using a Rolfs and Rodney type expression and parameter values from Table IV.

Case		g_1	g_2	χ^2_ν	$S^{TH}(0)$ (MeV b)	g_2/g_1	γ_{2p}/γ_{1p}
ZP21	A	0.507	0.921	0.978	59	1.81	1.00
	B	0.507	0.776	0.835	59	1.53	1.00
	C	0.507	0.772	0.855	59	1.52	0.97
	D	0.508	2.461	1.041	59	4.85	2.57
R71	A	0.502	0.912	0.774	63	1.82	0.99
	B	0.506	0.780	0.797	59	1.54	0.99
	C	0.506	0.775	0.787	60	1.53	0.97
	D	0.506	1.314	0.825	58	2.60	1.80
R32	A	0.509	0.934	1.081	57	1.84	1.01
	B	0.508	0.788	0.909	57	1.55	1.00
	C	0.508	0.784	0.935	57	1.54	0.98
	D	0.505	1.148	0.672	57	2.27	1.77

La Cognata *et al.* did not allow the feeding amplitudes to vary in fitting their own data with their Eq. (34), but took $g_1 = \gamma_{1p}$ and $g_2/g_1 = M^0_{21} = 1.13$; with these restrictions for case C, we find, for the three cases ZP21, R71, and R32, $\chi^2_\nu = 1.32$, 1.24, and 1.49, and $S^{TH}(0) = 68$, 70, and 66 MeV b. These are to be compared with La Cognata *et al.*'s values, for $a_p = 5.0$ fm, of $\chi^2_\nu = 1.9$ and $S^{TH}(0) = 65.0$ MeV b.

IV. CONCLUSION

The differences between the ratio of the feeding amplitudes and the ratio of the $^{15}N + p$ reduced-width amplitudes that are obtained in our best fits to the direct and THM data, as shown in Tables I and III for the R-matrix fits and in Tables IV and V for the Rolfs and Rodney type fits, indicate that the THM "S factor" is indeed a different quantity from the normal S factor obtained from direct measurements. The La Cognata *et al.* [1] peak is appreciably broader than that of either Zyskind and Parker [3] or Redder *et al.* [4]; because of this, even though La Cognata *et al.* normalized their data to the direct data in the resonant region $E_{c.m.} = 200$–400 keV, the maximum value of $S^{TH}(E)$ found by La Cognata *et al.* is about 3440 MeV b, while the direct measurements give about 3930 MeV b [3] and 3950 MeV b [4].

Our best R-matrix fits give $S(0) \approx 80$ MeV b, from both the Zyskind and Parker data [3] and the Redder *et al.* data [4]. The values of about 60 MeV b obtained from fits to the THM data [1] should not be regarded as values of the same quantity. More generally, in other cases where two or more resonances contribute significantly to the low-energy cross section, it seems that the THM would not necessarily give direct information about the value of $S(0)$.

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