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^{15}N(p, γ_0)^{16}O S factor

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Experimental values of the astrophysical S factor for the ^{15}N(p,γ_0)^{16}O reaction are fitted using both an R-matrix formula and an alternative formula, and values of the zero-energy S factor are derived.

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I. INTRODUCTION

In stars burning hydrogen through the CNO cycles, the relative abundances of the oxygen isotopes depend on the ratio of the ^{15}N(p,α_0)^{12}C and ^{15}N(p,γ_0)^{16}O cross sections at low energies [1]. The ^{15}N(p,α_0)^{12}C S factor has recently been discussed [2], suggesting a value of the zero-energy S factor $S(0)$ that is somewhat higher than the 65(7) MeV b adopted in the NACRE compilation [3]. Here we consider the S factor for the ^{15}N(p,γ_0)^{16}O reaction. The compilation [3] mentions two measurements, one by Hebbard [4] and the other by Rolfs and Rodney [5] (hereafter RR). The compilation adopts RR's value $S(0) = 64(6)$ keV b, saying that Hebbard's data are in disagreement with RR and with the resonance data of Refs. [6] and [7]. Hebbard [4] had given $S(E_p = 25\ \text{keV}) = 32$ keV b, equivalent [5] to $S(0) = 26$ keV b.

Hebbard [4] used the two-level approximation of standard R-matrix theory [8] to fit his own and other [9] ^{15}N(p,γ_0)^{16}O data, in addition to ^{15}N(p,α_0)^{12}C and ^{15}N($p,\alpha_1\gamma$)^{12}C data. The two levels are the 1^- levels of ^{16}O at excitation energies of 12.44 and 13.09 MeV ($E_p = 333$ and 1027 keV) [10]. RR used a different formula that included, in addition to contributions from the two levels and the interference between them, also a coherent contribution from direct capture, which was necessary to get a good fit to their data. The strength of the direct-capture contribution indicated a proton spectroscopic factor $C^2S = 1.8$ for ^{16}O(g.s.).

Standard R-matrix formulas, as used by Hebbard, are not in general justified for reactions involving photons. For such reactions a modified R-matrix formula that includes channel contributions has been given [11] for the case of electric-multipole transitions. The nonresonant part of the channel contributions is often referred to as hard-sphere capture, or direct capture, such as RR found to be necessary to fit their data.

Here we fit the RR data and the Hebbard data, using both the modified R-matrix formula and the RR formula. The data are summarized in the next section, while Sec. III gives the formulas and some of the approximations that have been made. The fits are given in Sec. IV, and these and other results are discussed in Sec. V.

II. DATA

RR's Fig. 3 gives their measured values of the ^{15}N(p,γ_0)^{16}O S factor with uncertainties. We fit the data only for $E_p < 1400$ keV, to avoid contributions from the 1^+ resonance at $E_p = 1640$ keV and because we assume that only s-wave protons are contributing. To obtain the absolute cross section for the ^{15}N(p,γ_0)^{16}O reaction, RR used their observed intensity of these γ rays relative to those from the ^{15}N($p,\alpha_1\gamma$)^{12}C reaction and the value $\sigma = 250 \pm 35$ mb [12] for the latter reaction at the $E_p = 1210$ keV resonance. They found the ^{15}N(p,γ_0)^{16}O cross section at the upper peak to be $420 \pm 60\ \mu$b. Criticism [13] of RR's ^{15}N($p,\alpha_1\gamma$)^{12}C cross section near the 1210 keV peak suggests some extra uncertainty in their absolute cross section. In any case, it is difficult to see how RR can obtain $S(0)$ to better than $\pm14\%$.

Figure 4 of Hebbard [4] shows his measured ^{15}N(p,γ_0)^{16}O cross section values for proton energies up to 627 keV. His two-level R-matrix fit included higher energy values from Hagedorn [9], normalized to 700 μb at the upper peak. In our fits we use Hebbard's data for $E_p = 240$–627 keV, together with values taken from Fig. 8 of Hagedorn [9] for $E_p = 825$–1300 keV (with the background indicated in Fig. 8 subtracted), but now normalized to a peak cross section of 610 μb, following the revision in Ref. [14]. Hebbard's Fig. 4 shows points at lower energies, but he rejects those for $E_p < 220$ keV because they are affected by boron contamination. Hebbard does not give uncertainties on his values. Somewhat arbitrarily, we assume 10% uncertainties, with 20% for points between 425 and 550 keV, as Hebbard says that less weight should be attached to these points than to others. We also take 20% uncertainty on the Hagedorn data [14]. We refer to this set of data, converted to values of the S factor, as HH. Because of the uncertain uncertainties, values of χ^2 from HH fits should not be compared with those from RR fits.

From the peak cross sections, values of radiative widths have been obtained [4,5]. These are thoroughly discussed and compared with values from other reactions in Ref. [14].

From stripping reactions, such as ^{15}N(^{3}He, d)^{16}O, the spectroscopic factor for the ^{16}O(g.s.) is 3.1 [10], or $C^2S = 1.55$.

III. FORMULAS AND APPROXIMATIONS

The R-matrix formulas with channel contributions are taken from Ref. [11]. We use the two-level approximation and, in addition to the proton and γ_0 channels, we also include contributions from the α_0 channel; α_1 contributions are expected to be negligible [10]. In the notation of Ref. [11], we have $J_f = 0$, $J_i = 1$, $s_e = 1$, $l_i = 0$, $l_f = 1$, and $L = 1$.

For given values of the channel radii a_c $(c = p, \alpha)$ and boundary condition parameters B_c, the adjustable parameters are the eigenenergies of the two levels $E_\lambda (\lambda = 1, 2)$, their reduced-width amplitudes $\gamma_{\lambda c}$ in the proton and α_0 channels, the internal photon reduced-width amplitudes $\gamma_{\lambda\gamma}$(int), and the dimensionless reduced-width amplitude θ_f for ^{16}O(g.s.) $\to$ ^{15}N(g.s.) $+ p$. In addition to the usual R-matrix penetration factor P_c and shift factor S_c, the formulas involve radial integrals J' and J'', which are known functions of a_p and E. As in Ref. [2], we use the conventional value $a_p = 5.03$ fm and $a_\alpha = 6.5$ fm. So that parameter values from the ^{15}N$(p, \alpha_0)^{12}$C analysis in Ref. [2] can be used, we take $B_c = S_c(E_2)$.

The spectroscopic factor C^2S for ^{16}O(g.s.) $\to$ ^{15}N(g.s.) $+$ p can be obtained from

$$\theta_f^2 = C^2 S\, \theta_{\rm sp}^2, \tag{1}$$

where

$$\theta_{\rm sp}^2 = \frac{a_p}{2} \frac{u^2(a_p)}{\int_0^{a_p} u^2(r) {\rm d}r}. \tag{2}$$

With the ^{15}N(g.s.) $+p$ radial wave function $u(r)$ calculated in a central Woods–Saxon potential with radius parameter $r_0 =$ 1.25 fm and diffuseness $a = 0.65$ fm, we find $\theta_{\rm sp}^2 = 0.0598$ for $a_p = 5.03$ fm. It is small because of the large binding energy (12.128 MeV).

The RR formula is given in Eq. (9) of Ref. [5], together with Eqs. (5) and (8). RR used compilation [15] values for the energies and partial widths of the two resonances ($E_1 = 0.317$ MeV, $\Gamma_{1p} = 1.1$ keV, $\Gamma_{1\alpha} =$ 98 keV, $E_2 = 0.963$ MeV, $\Gamma_{2p} = 100$ keV, $\Gamma_{2\alpha} = 45$ keV). They took the energy dependence of the direct-capture S factor $S_{\rm DC}(E)$ from Rolfs [16] and adjusted its magnitude to best fit the data off the resonances; this determined the value of C^2S. RR used approximate expressions for the energy dependence of the proton and α-particle partial widths; they imply penetration factors that vary more rapidly with energy than do the R-matrix quantities with $a_p = 5.03$ fm and $a_\alpha = 6.5$ fm.

IV. FITS

In fits involving the R-matrix formulas, we initially use values of the E_λ and $\gamma_{\lambda c}$ obtained [2] in fits to ^{15}N$(p, \alpha_0)^{12}$C data. The three remaining adjustable parameters are $\gamma_{\lambda\gamma}$(int) and θ_f. Table I gives the results of these fits to either the data of RR or HH, when the parameter values R32 A, B, or C in Table I of Ref. [2] are used for E_λ and $\gamma_{\lambda c}$ (these are from fits to the ^{15}N$(p, \alpha_0)^{12}$C data of Redder *et al.* [7]). In place of θ_f we give values of the equivalent quantity C^2S, and we also give values of the reduced $\chi^2 (\chi^2_\nu = \chi^2$/degree of freedom) and of $S(0)$. In some fits, C^2S is fixed at 1.55 [10]. If ZP21 values are used (from the Zyskind and Parker [17] ^{15}N$(p, \alpha_0)^{12}$C data), χ^2_ν and $S(0)$ are little changed. With R71 values, χ^2_ν is significantly increased but $S(0)$ is about the same. In Table II we give results of fits in which all nine parameters are allowed to vary and when eight parameters are varied with θ_f fixed to make $C^2S = 1.55$. Based on the values in Table II, Table III gives the resonance energy and partial observed widths of the lower level, obtained by transforming to $B_c = B'_c = S_c(E'_1)$, and also the partial widths of the upper level.

Tables IV–VI are similar to Tables I–III, except that the RR formula is used instead of the R-matrix formula. The penetration factors are the same as those in Tables I–III. In Table IV, the values of E_λ and $\gamma_{\lambda c}$ are chosen to fit the values of the level energies and partial widths used by RR ($\gamma_{1p} = 0.465$ MeV$^{1/2}$, $\gamma_{1\alpha} = -0.113$ MeV$^{1/2}$, $\gamma_{2p} =$ 0.485 MeV$^{1/2}$, $\gamma_{2\alpha} = 0.0723$ MeV$^{1/2}$), and only $\gamma_{\lambda\gamma}$ and θ_f are varied. In Table V, the values of E_λ and $\gamma_{\lambda c}$ are also allowed to vary.

The RR data are shown in Fig. 1, with three fits, corresponding to the parameter values given in the first row in Table I, the second row in Table II, and the first row in Table IV. Figure 2 gives the HH data, with three fits: the fifth row in Table I, the fourth row in Table II, and the fourth row in Table IV.

TABLE I. R-matrix fits to ^{15}N$(p, \gamma_0)^{16}$O S-factor data [4,5,9], with level parameters E_λ and $\gamma_{\lambda c}$ from Ref. [2].

Data	Case		$\gamma_{1\gamma}$ (int)	$\gamma_{2\gamma}$ (int)	C^2S	χ^2_ν	$S(0)$ (keV b)
RR	R32	A	0.0001	0.098	29.5	4.68	55.0
		A	0.144	0.245	1.55[a]	7.65	34.9
		B	0.050	0.158	14.7	4.75	50.3
		C	0.040	0.154	17.1	4.76	51.0
HH	R32	A	0.086	0.227	5.82	1.86	39.2
		A	0.137	0.279	1.55[a]	1.98	34.3
		B	0.124	0.272	2.32	1.77	36.0
		B	0.137	0.285	1.55[a]	1.75	34.8
		C	0.116	0.270	2.87	1.76	36.3

[a]Fixed value.

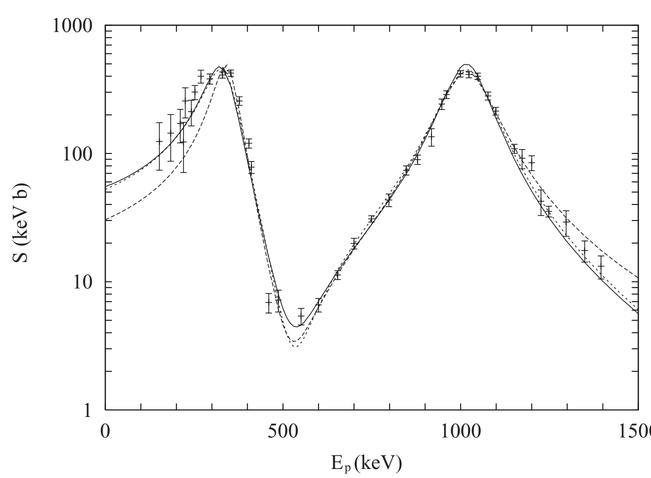

FIG. 1. The ^{15}N$(p, \gamma_0)^{16}$O S factor as a function of proton energy. The experimental points are from Rolfs and Rodney [5]. The solid and dashed curves are R-matrix fits, with parameter values given in Table I, row 1 (solid) or Table II, row 2 (dashed). The dotted curve is a fit using the Rolfs and Rodney formula, with parameter values given in Table IV, row 1.

TABLE II. R-matrix fits to $^{15}N(p, \gamma_0)^{16}O$ S-factor data [4,5,9], with level parameters variable. [E_λ in MeV, $\gamma_{\lambda c}$ in MeV$^{1/2}$, $S(0)$ in keV b.]

Data	Case	E_1	γ_{1p}	$\gamma_{1\alpha}$	$\gamma_{1\gamma}$(int)	E_2	γ_{2p}	$\gamma_{2\alpha}$	$\gamma_{2\gamma}$(int)	C^2S	χ^2_ν	$S(0)$
RR	a	0.291	0.306	−0.116	0.195	0.984	0.645	−0.0285	−0.022	63.2	3.16	62.4
	b	0.134	0.717	−0.092	0.121	0.964	0.570	0.0734	0.245	1.55[a]	5.13	30.4
HH	c	0.183	0.596	−0.103	0.085	0.960	0.515	0.0735	0.237	5.41	1.28	35.2
	d	0.183	0.585	−0.100	0.128	0.956	0.490	0.0829	0.297	1.55[a]	1.30	30.7

[a]Fixed value.

TABLE III. Resonance energy and partial widths corresponding to fits in Table II.

Data	Case	E'_1 (MeV)	Γ^0_{1p} (keV)	$\Gamma^0_{1\alpha}$ (keV)	Γ^0_1 (keV)	Γ^0_{2p} (keV)	$\Gamma^0_{2\alpha}$ (keV)	Γ^0_2 (keV)
RR	a	0.325	0.26	92.5	92.8	143	5.6	148
	b	0.324	1.27	71.0	72.3	116	39.1	155
HH	c	0.320	1.00	85.9	86.9	98	40.3	138
	d	0.318	1.00	83.4	84.4	90	51.9	142

TABLE IV. Fits to $^{15}N(p, \gamma_0)^{16}O$ S-factor data [4,5,9] using RR formula. The values of E_λ and $\gamma_{\lambda c}$ are chosen to fit the level energies and partial widths used by RR.

Data	$\gamma_{1\gamma}$	$\gamma_{2\gamma}$	C^2S	χ^2_ν	$S(0)$ (keV b)
RR	0.149	0.280	0.61	3.73	52.4
	0.157	0.266	1.55[a]	5.59	68.0
	0.158	0.263	1.8[a]	6.47	71.4
HH	0.127	0.314	0.23	1.91	40.7
	0.129	0.269	1.55[a]	4.09	58.3

[a]Fixed value.

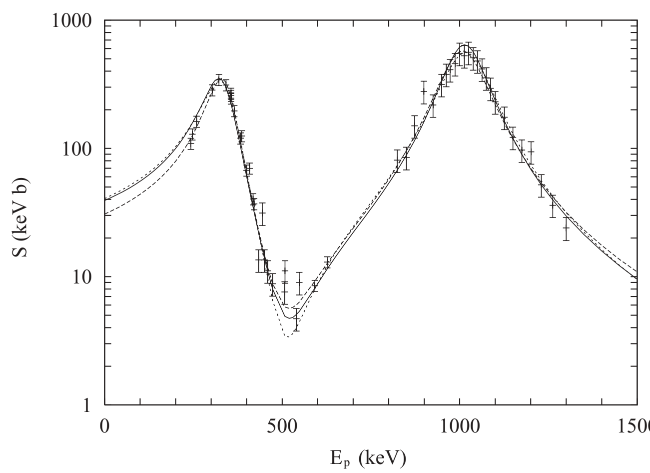

FIG. 2. The $^{15}N(p, \gamma_0)^{16}O$ S factor as a function of proton energy. The experimental points are from Hebbard [4] ($E_p \leqslant 627$ keV) and from Hagedorn [9] ($E_p \geqslant 825$ keV) normalized to a peak cross section of 610 μb [14]. The solid and dashed curves are R-matrix fits, with parameter values given in Table I, row 5 (solid) or Table II, row 4 (dashed). The dotted curve is a fit using the Rolfs and Rodney formula, with parameter values given in Table IV, row 4.

TABLE V. Fits to $^{15}N(p, \gamma_0)^{16}O$ S-factor data [4,5,9] using RR formula with E_λ and $\gamma_{\lambda c}$ allowed to vary. [E_λ in MeV, $\gamma_{\lambda c}$ in MeV$^{1/2}$, $S(0)$ in keV b.]

Data	Case	E_1	γ_{1p}	$\gamma_{1\alpha}$	$\gamma_{1\gamma}$	E_2	γ_{2p}	$\gamma_{2\alpha}$	$\gamma_{2\gamma}$	C^2S	χ^2_ν	$S(0)$
RR	e	0.323	0.458	−0.107	0.140	0.976	0.584	0.0025	0.236	0.72	3.50	49.7
	f	0.321	0.129	−0.112	0.537	0.981	0.567	0.0000	0.232	1.55[a]	3.98	64.3
	g	0.321	0.106	−0.113	0.665	0.982	0.562	0.0004	0.231	1.8[a]	4.35	67.9
HH	h	0.324	1.578	−0.100	0.035	0.965	0.572	−0.0041	0.254	0.046	1.27	30.1
	i	0.328	0.049	−0.115	1.123	0.977	0.493	0.0412	0.265	1.55[a]	2.26	52.8

[a]Fixed value.

TABLE VI. Partial widths corresponding to fits in Table V.

Data	Case	Γ_{1p} (keV)	$\Gamma_{1\alpha}$ (keV)	Γ_1 (keV)	Γ_{2p} (keV)	$\Gamma_{2\alpha}$ (keV)	Γ_2 (keV)
RR	e	1.01	87.3	88.3	145	0.1	145
	f	0.08	96.0	96.1	137	0.0	137
	g	0.05	97.6	97.7	134	0.0	134
HH	h	12.0	77.1	89.1	139	0.1	139
	i	0.01	101.1	101.1	103	14.6	118

V. DISCUSSION

R-matrix fits to the RR data, given in Table I, have C^2S values much larger than the experimental value of 1.55, and $S(0) \approx$ 50–55 keV b. A better fit is obtained if all the parameter values are allowed to vary (Table II, case a), but C^2S is even larger and the values of Γ^0_{1p} and $\Gamma^0_{2\alpha}$ are unacceptably low (Table III). With the restriction $C^2S = 1.55$, all partial widths are reasonable and $S(0)$ is reduced to 30–35 keV b, while χ^2_ν is increased significantly.

The R-matrix fits to the HH data have smaller C^2S values, and the restriction $C^2S = 1.55$ does not increase χ^2_ν significantly. All fits have reasonable partial widths, with $S(0) \approx 35$ keV b.

Fits to the RR data using the RR formula and their energy and width values (Table IV) give C^2S small and $S(0) \approx$ 50 keV b. With $C^2S = 1.55$, χ^2_ν increases and so does $S(0)$. With $C^2S = 1.8$, as used by RR, χ^2_ν and $S(0)$ are larger still; this fit should be closest to the fit that RR made, and our $S(0) = 71.4$ keV b is reasonably close to their 64 keV b. With level energies and widths allowed to vary (Table V), better fits are obtained with smaller values of $S(0)$, but all have unacceptably small values of $\Gamma^0_{2\alpha}$ (Table VI), and those for $C^2S = 1.55$ or 1.8 have values of Γ^0_{1p} that are much too small.

Fits to the HH data using the RR formula (Table IV) give very small values of C^2S and $S(0) \approx 40$ keV b; with $C^2S =$ 1.55, χ^2_ν is much increased and $S(0) \approx 60$ keV b. The same is true when the level parameters are allowed to vary (Table V), but the values of Γ^0_{1p} and $\Gamma^0_{2\alpha}$ are unacceptable.

Some of the difference between the R-matrix fits and the RR-formula fits can be attributed to the different "single-particle" direct-capture contributions. With $C^2S = 1$ (and all $\gamma_{\lambda c} = 0$), the R-matrix intensity is only about 5% of the RR intensity. The forms of the formulas are also somewhat different, as discussed in Ref. [2].

It seems that, for acceptable values of the partial level widths, the RR data are best fitted using the RR formula (with a smaller direct-capture contribution than was found by RR) with $S(0) \approx 50$ keV b, while the best fit using the R-matrix formula is not as good, but has $S(0) \approx 30$ keV b. On the other hand, the HH data are equally well fitted using the R-matrix formula with $S(0) \approx 35$ keV b, and with the RR formula (with small direct-capture contribution), with $S(0) \approx 40$ keV b.

The NACRE compilation [3] rejected the Hebbard data on the basis that they are in disagreement with the RR data and with other data [6,7]. The figure in Ref. [3] for $^{15}\mathrm{N}(p,\gamma_0)^{16}\mathrm{O}$ suggests that the main difference between the Hebbard and RR data is at low energies, but this involves the points that Hebbard says should be disregarded because of $^{11}\mathrm{B}$ contamination (and they are omitted in the present analysis). The $^{15}\mathrm{N}(p,\gamma_0)^{16}\mathrm{O}$ measurements of Schardt, Fowler, and Lauritsen [6] are for $E_p \geqslant 860$ keV, well above the Hebbard range. Hebbard [4] discusses the $^{15}\mathrm{N}(p,\alpha_0)^{12}\mathrm{C}$ and $^{15}\mathrm{N}(p,\alpha_1\gamma)^{12}\mathrm{C}$ measurements of Schardt, Fowler, and Lauritsen [6] and argues for renormalization of some of them, but this hardly constitutes disagreement. Redder *et al.* [7] measured only the $^{15}\mathrm{N}(p,\alpha_0)^{12}\mathrm{C}$ cross section. The resonance parameters they used for the lower 1^- level [$\Gamma_{1p} =$ 0.9(1) keV, $\Gamma_{1\alpha} = 102(4)$ keV] are not too different from those found in our fits to the Hebbard data.

It seems that the Hebbard $^{15}\mathrm{N}(p,\gamma_0)^{16}\mathrm{O}$ data should not be neglected. With the modified R-matrix formula, which has more *a priori* justification than the RR formula, the fits to the HH data appear to be more consistent with other data involving the 12.44 and 13.09 MeV levels of $^{16}\mathrm{O}$ than are the fits to the RR data. They suggest $S(0) \approx 35$ keV b, appreciably less than the compilation [3] value of 64(6) keV b. With the revised value for the $^{15}\mathrm{N}(p,\alpha_0)^{12}\mathrm{C}$ reaction of $S(0) \approx 80$ MeV b [2], the ratio becomes about 1/2300, rather than the value 1/880 recommended by RR, or 1/1000 from the values in the NACRE compilation [3].

After this work was completed, an article was published [18] in which the $^{15}\mathrm{N}(^3\mathrm{He},d)^{16}\mathrm{O}$ reaction is used to measure the asymptotic normalization coefficient (ANC) for the $^{16}\mathrm{O}$ ground state, and this determines the strength of the direct-capture contribution in an R-matrix calculation of the $^{15}\mathrm{N}(p,\gamma_0)^{16}\mathrm{O}$ S factor. This ANC corresponds to a spectroscopic factor $C^2S = 2.10$ [18], compared with 1.55 [10] used in some of our fits. The direct-capture contribution is much smaller than that which RR had used, as is also found here, leading to $S(0) = 36.0 \pm 6.0$ keV b [18].

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