Low-lying states in the unbound $^{11}$N nucleus


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We have investigated the low-energy states of $^{11}$N by the resonant elastic-scattering method in inverse kinematics using a $^{10}$C beam and a $(\text{CH}_3)_n$ target at the CYCLONE facility at Louvain-la-Neuve. Recoil protons were detected at laboratory angles $\theta_{\text{lab}} = 5.2^\circ \pm 18.3^\circ$ in a $\Delta E-E$ detector system. The absolute $^{10}$C+$^p$ elastic cross-section data were analyzed in the $R$-matrix framework. We found $^{11}$N to be unbound with respect to proton emission by 1.54 $\pm$ 0.02 MeV, with a decay width of 0.83 $\pm$ 0.03 MeV. These results are used to calculate the two-proton decay width of the $^{12}$O ground state.

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I. INTRODUCTION

The properties of light exotic nuclei are a major source of interest in nuclear physics research [1]. Ab initio calculations have now been applied to all nuclei up to mass number $A = 10$ [2] and have revealed important insights such as the need for including three-body interaction terms to reproduce energy level structures. The properties of $A = 11$ nuclei will represent a key challenge for future calculations. Members of the $A = 11$ isobaric chain have already produced startling new insights into nuclear structure. The $^{11}$Li nucleus provided the first evidence for the (two-)neutron halo [3,4]. It exhibits a two-proton decay width of the ground state of $^{12}$O [7–10]. Much experimental effort has been devoted to elucidating the low-energy resonances of $^{11}$N [11–16], but there remains considerable disagreement between experimental results and with theoretical predictions [5,6,17–19], particularly with respect to the ground state.

The present paper presents a new, high-precision study of the low-energy resonances of $^{11}$N. In Sec. II, the experimental method is described. The data analysis is presented in Sec. III. In Sec. IV, a discussion of the results and a comparison with previous results are presented. Some conclusions are given in Sec. V.

II. EXPERIMENTAL METHOD

We have used the technique of elastic scattering in inverse kinematics [20] to investigate the $^{11}$N nucleus. An isobarically pure $^{10}$C beam was produced at the CYCLONE radioactive beam facility for the first time, specifically for this experiment. The $^{10}$C atoms were produced through the $^{10}$B$(p,n)^{10}$C reaction by bombarding a natural boron nitride target with an intense 30 MeV proton beam produced by the CYCLONE30 cyclotron, and these were then ionized to the $2^+$ state in an Electron Cyclotron Resonance (ECR) source before being post-accelerated by the CYCLONE110 cyclotron. The $^{10}$C beam was stopped 1.5 m downstream from the target in a Faraday cup equipped with a current amplifier suitable to work with low-beam current and with an electron-suppression system. The averaged intensity on target was typically $3 \times 10^5$ pps.

We used beams of $E_{\text{lab}} = 25.5$ and 32 MeV to bombard a 3 mg/cm$^2$ polyethylene $(\text{CH}_3)_n$ foil. The c.m. energy range covered by the present experiment was $E_{c.m} = 0.7$–2.8 MeV, with respect to the $^{10}$C+$^p$ threshold, allowing the study of the $1/2^+$ and $1/2^-$ states in $^{11}$N. The target thickness was chosen such that the expected full width of the $1/2^+$ ground state could be encompassed at a single beam energy, while limiting the total energy straggling of the beam in the target. Figure 1 shows a schematic layout of the experimental setup. The recoil protons were detected using a compact disk silicon strip detector based $\Delta E-E$ telescope array called CD-PAD which allows for very clean separation between proton, $\alpha$, and $\beta$ particles [21]. No signals indicating heavier ions were observed in the $\Delta E-E$ telescope. The beam energy was cross-checked with the measured value of the most energetic proton observed. The nominal and measured laboratory energies...
agreed within $\pm 100$ keV (better than 0.4%), and the total beam energy spread was less than 100 keV full width at half maximum (FWHM). The CD-PAD array was situated 12.1 cm downstream from the target, covering a laboratory angular range of $\theta_{\text{lab}} = 4.6^\circ - 18.9^\circ$ in 16 strips. The energy calibration of the detector array was performed by means of a three-line $\alpha$ source ($^{239}$Pu, $^{241}$Am, $^{244}$Cm) and a precision pulser. The laboratory proton energy resolution was 55 keV for the $\Delta E$ detectors and 90 keV for the PAD detectors. The combined energy resolution was $\delta E_p = 105$ keV. Compared with earlier $^{10}$C+$p$ studies [12,14], the present improved setup allows better energy and angular resolutions and, because the beam energy is lower and the target thinner, probably a smaller energy straggling. On the other hand, the energy range investigated is smaller here, as we concentrate mainly on the properties of the $1/2^+ \ ^{11}$N ground state.

Two important effects have to be taken into account in the data analysis. First, the opening angle of the detector strips introduces an uncertainty in the proton energy given by $\delta E_0 = 2 E_p \tan \theta_{\text{lab}} \Delta \theta_{\text{lab}}$ [22]. Typical values range between 20 and 160 keV (for the proton energies and angles covered). The second effect is the straggling of the beam particles and of the recoil protons in the target. The most important straggling effect is that of the beam in the target, which produces an additional uncertainty in the laboratory energies of the recoil protons, typically of less than 6%. The total energy broadening is obtained by adding quadratically all contributions, and it is one of the inputs in the theoretical analysis of the cross section (Sec. III).

Figure 2 shows a raw $\Delta E$-$E$ spectrum obtained at the beam energy of 25.5 MeV (spectra for 32 MeV are similar). Recoil protons from the $^{10}$C+$p$ elastic scattering are clearly separated from $\alpha$ particles resulting from reactions of $^{10}$C with the C content in the target. This spectrum shows that the use of polyethylene targets is perfectly suitable for this kind of experiment provided the appropriate detector system is used. A detailed simulation of the proton spectra was performed including the energy loss [23] and the energy straggling of the $^{10}$C beam and the protons in the target. The pulse height defect effect due to the different energy deposition in the detector by protons with respect to $\alpha$ particles (below 1%) [24] as well as the beam energy spread were also included.

From the recoil proton spectra, we obtained absolute differential cross sections for 10 effective laboratory angles (recoil spectra of the 12 innermost strips were added two by two) in the range $\theta_{\text{lab}} = 5.2^\circ - 18.3^\circ$ and for c.m. energies $E_{\text{c.m.}} = 0.7-2.75$ MeV by correcting the number of counts for the solid angle of the detectors ($\pm 5\%$), the H content in the target ($\pm 10\%$), and the total number of incident beam particles ($\pm 15\%$). This last quantity is the most critical one to obtain an absolute normalization. We used the Rutherford elastic scattering of the $^{10}$C beam in a Au target to calibrate the current amplifier at the Faraday cup. Charge-state distribution was calculated based on the method described in Ref. [25]. This technique has been successfully used in several experiments (see, for example, Ref. [26]). Figure 3 shows the cross section as a function of the c.m. energy for three typical angles $\theta_{\text{c.m.}} = 143.5^\circ$, 158.5$^\circ$, and 169.7$^\circ$ [27].

III. DATA ANALYSIS

A. Resonance parameters

We used the $R$-matrix model [28] in the one-channel, one-level approximation to fit the differential cross sections in the $143.5^\circ-169.7^\circ$ c.m. angular range and 0.7–2.75 MeV c.m. energy range ($N = 398$ data points). In the $R$-matrix framework, the nuclear phase shift is defined by

$$\delta^\ell = \delta_{\text{HS}}^\ell + \delta_R^\ell,$$

where $\delta_{\text{HS}}^\ell$ is the hard-sphere phase shift and $\delta_R^\ell$ is the resonant phase shift given by

$$\delta_R^\ell = \arctan \frac{P_t R^\ell}{1 - S_t R^\ell},$$

where $P_t$ and $S_t$ are the penetration and shift factors, respectively (we have assumed the boundary condition $B_\ell = 0$). The $R$-matrix $R^\ell$ defined in the one-level approximation is given by

$$R^\ell = \frac{\gamma^\ell}{E^\ell - E},$$

with $\gamma^\ell$ and $E^\ell$ the formal reduced width amplitude and the energy of the $R$-matrix pole, respectively.
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Here, $\ell$ values up to $\ell_{\text{max}} = 3$ have been included. We have included resonance contributions for partial waves $\ell$, $J = 0, 1/2; 1, 1/2; 2, 5/2$ and used hard-sphere phase shifts for other $\ell J$ values (see Ref. [30] for more details). The fitted parameters are the pole parameters $E_\ell$, $\Gamma_\ell$ (“calculated” parameters) converted to the resonance energies $E_r$, and proton widths $\Gamma_r$ (“observed” parameters) of the $^{11}$N states [31,32]. In order to account for the experimental energy spread described above, we convoluted the calculations at each angle with a Gaussian energy distribution, with a FWHM set equal to the total experimental energy broadening at that angle (c.m. values ranging from 120 to 160 keV). For resonances with large widths, the resulting parameters depend slightly on these values; only the $\chi^2$ values depend.

The fits are performed with different values of the $R$-matrix channel radius $a = 5.5–7.0$ fm. Because the data do not extend to the energy of the 5/2$^+$ state, we fixed its resonance energy and width at 3.70 $\pm$ 0.05 and 0.55 $\pm$ 0.05 MeV, respectively. These are the weighted averaged values of Table IV in Ref. [14] and Table II in Ref. [16]. The remaining free parameters are the resonance energies and decay widths of the 1/2$^+$ and 1/2$^-$ states ($E_0$, $\Gamma_0$, $E_1$, $\Gamma_1$). The influence of the absolute normalization, the choice of the channel radius $a$, the uncertainties of the 5/2$^+$ state parameters, and the uncertainty associated with the experimental energy spread were carefully studied by performing $R$-matrix fits for different initial conditions. For each fit, the uncertainties in the fitted parameters are calculated from the $R$-matrix fits that have $\chi^2 = \chi^2_{\text{min}} + 1$. Table I shows the fits for different values of the channel radius $a$. Similar results are obtained when the normalization, the parameters of the 5/2$^+$ state, and the energy spread are varied. We have found that all parameters depend only slightly on these conditions, with the exception of $\Gamma_1$. Figure 4 shows the variation of $\chi^2$ with a normalization factor $\xi = 1.0$ and of $\xi$ for the best $\chi^2$ values) as a function of $a$. From the overall study of the effect of the different uncertainties (absolute normalization, value of the channel radius, parameters of the 5/2$^+$ state, and experimental energy resolution), we recommend the following values for the parameters of the 1/2$^+$ and the

| $a$ (fm) | $E_0$ (MeV) | $\Gamma_0$ (MeV) | $E_1$ (MeV) | $\Gamma_1$ (MeV) | $\chi^2_{\text{min}}$ | 5.5 | 1.560 $\pm$ 0.002 | 0.831 $\pm$ 0.007 | 2.220 $\pm$ 0.004 | 0.83 $\pm$ 0.02 | 430 |
| 5.75 | 1.552 $\pm$ 0.002 | 0.828 $\pm$ 0.006 | 2.236 $\pm$ 0.004 | 0.93 $\pm$ 0.02 | 322 |
| 6.0 | 1.546 $\pm$ 0.002 | 0.827 $\pm$ 0.007 | 2.255 $\pm$ 0.003 | 1.05 $\pm$ 0.02 | 251 |
| 6.25 | 1.540 $\pm$ 0.002 | 0.827 $\pm$ 0.006 | 2.277 $\pm$ 0.003 | 1.18 $\pm$ 0.02 | 231 |
| 6.5 | 1.535 $\pm$ 0.002 | 0.828 $\pm$ 0.007 | 2.301 $\pm$ 0.004 | 1.32 $\pm$ 0.02 | 287 |
| 6.75 | 1.530 $\pm$ 0.002 | 0.830 $\pm$ 0.007 | 2.329 $\pm$ 0.004 | 1.49 $\pm$ 0.02 | 430 |
| 7.0 | 1.528 $\pm$ 0.002 | 0.840 $\pm$ 0.007 | 2.366 $\pm$ 0.004 | 1.67 $\pm$ 0.02 | 685 |

The observed resonant width, which should be compared with the experimental width, is given by

$$\Gamma_{r\ell} = \frac{\Gamma_\ell}{1 + \frac{\delta_\ell^2}{r_1^2} (dS_i/dE)_{E_\ell}},$$

(4)

where $E_{r\ell}$ is the resonance energy defined as the energy at which $\delta_\ell^2$ passes through $\pi/2$, and $\Gamma_\ell$ is the formal width of the $R$-matrix pole defined by

$$\Gamma_\ell = 2\gamma_\ell^2 P_\ell(E_{r\ell}).$$

(5)

For broad resonances, as expected in $^{11}$N, the peak energy $E_m$ and the peak width $\Gamma_m$ are also used to characterize the states differently from the resonant energy $E_{r\ell}$ and resonant width $\Gamma_{r\ell}$. The peak energy is the energy where the density-of-states function $\rho$ reaches the maximum, with $\rho = \sin^2 \delta_\ell^R / P_\ell$ [29]; the peak width is defined as the FWHM of the peak.
1/2− states: \( E_{r0} = 1.54(2), \Gamma_{r0} = 0.83(3), E_{r1} = 2.27(5), \) and \( \Gamma_{r1} = 1.15(25) \) MeV.

The best fit of Table I \((a = 6.25 \text{ fm}, \chi^2 = 231)\) is shown as thick solid curves in Fig. 3. To prove the sensitivity of the fits, we performed \( R \)-matrix calculations (no fits) with \( a, E_{r0}, \Gamma_{r0}, \) and \( E_{r1} \) fixed at their values of the best fit of Table I, but using different values for \( \Gamma_{r1} \). These calculations are shown in Fig. 3 for \( \Gamma_{r1} = 0.6 \) and for \( \Gamma_{r1} = 0.8 \) MeV. These curves are in clear disagreement with the data. Similar results are obtained by varying \( E_{r0}, \Gamma_{r0}, \) or \( E_{r1} \).

As an illustration, Table II gives a comparison of the experimental parameters, the formal parameters, and the peak parameters of the \( \ell = 0, 1 \) low-energy levels of \(^{11}\text{N}\) for typical values of the channel radius \( a \).

### B. Spectroscopic factors

In order to obtain values of the spectroscopic factors \( S_\ell \), we used [33,34]

\[
\Gamma_\ell = S_\ell \Gamma_{sp,\ell},
\]

### IV. DISCUSSION

#### A. Comparison with previous results on low-lying resonances in \(^{11}\text{N}\)

For more than 30 years, there has been considerable experimental and theoretical effort to study the low-energy states of \(^{11}\text{N}\). In the pioneering work of Benenson et al. [11], a light-ion multinucleon transfer reaction was used; a resonance found at 2.24 MeV was interpreted as the analog of the...
first excited state of $^{11}$Be. Axelsson et al. [12] reported evidence for the ground and first two excited states of $^{11}$N in experiments performed at Grand Accelerateur National d’Ions Lourds (GANIL) using a $^{10}$C beam produced by heavy-ion fragmentation to bombard an extended methane gas target. Later, the Axelsson et al. data were reanalyzed, along with similar data from Michigan State University (MSU), and significantly revised values were published by Markenroth et al. [14]. In particular, the ground state was reported at a resonance energy of 1.27 MeV with a decay width of 1.44 MeV. Lépine-Szily et al. used the transfer reaction $^{12}$C($^{14}$N,$^{15}$C)$^{11}$N at GANIL [13] and observed the first excited state at 2.18 MeV with a width of 0.44 MeV; no evidence of the ground state was observed. Heavy-ion transfer-reaction studies performed in the Spectromètre à Perte d’Energie du Ganil (SPEG) spectrometer at GANIL by Oliveira et al. [15] reported a 1/2$^+$ ground state with an energy of 1.63 MeV and a width of 0.4 MeV, in marked disagreement with Ref. [14]. Furthermore, while reasonable agreement was obtained for the energy of the 1/2$^-$ first excited state, there was considerable disagreement in its decay widths, 0.84 [14] and 0.25 MeV [15]. In the SPEG experiment, an additional state was reported at an energy of 3.06 MeV that was not observed in Ref. [14], although both experiments reported the observation of the predicted 5/2$^-$ second excited state around 3.7 MeV (width $\sim$0.5 MeV). Finally, Guimarães et al. investigated the transfer reaction $^{14}$N($^3$He,$^6$He)$^{11}$Na at RIKEN [16]. The reported $^{11}$N ground state energy was 1.31 MeV, in fair agreement with the work of [14], but the decay width of 0.24 MeV was the smallest value ever reported and in strong disagreement with several theoretical predictions [5,6,17–19] (see Table IV). Notice also that the definitions of “energy” and “width” may not be the same in all the prescriptions used to analyze the experimental results [6]. For example, the values reported by [6,12,14,16,18] are more likely peak energies and widths rather than resonance energies and widths. Table IV shows a summary of experimental and theoretical values, including the present work, for the low-lying 1/2$^+$ and 1/2$^-$ resonances in $^{11}$N. As stated above, uncertainties over previous definitions of measured quantities make direct comparisons difficult. However, we can note that the present precise value for the width of the ground state disagrees significantly with all previous results, while the resonance energy appears consistent with that obtained by Oliveira et al. [15]. For the 1/2$^-$ state a more consistent picture emerges for the resonance energies, with the present value being the most precise of the $^{10}$C+$p$ results. The width obtained here is significantly higher than other values and appears to be consistent only with the value reported by Markenroth et al. [14], both with comparable uncertainties.

The differences between theoretical calculations are even larger than the differences between experiments because of the very specific nature of $^{11}$Be and $^{11}$N, where the parity inversion is difficult to reproduce, as in Ref. [17]. Moreover, for broad resonances as in $^{11}$N, the definitions of energy and width may depend on the model. For example, Markenroth et al. [14] used the potential model for three levels plus a background term to analyze their $^{10}$C+$p$ data and different definitions for the energy and width of an unbound level. The present results are deduced from $^{10}$C+$p$ data, which should make a comparison with further models easier.

### B. Width of the $^{12}$O ground state

The contribution to the width of $^{12}$O(g.s.) due to sequential decay through the 1/2$^+$ $^{11}$N(g.s.) may be calculated as in Ref. [8]. It depends on the value of $E_{r0}$ for $^{11}$N and on the reduced width $\gamma_0^2$ for $^{12}$O(g.s.) $\rightarrow$ $^{11}$N(g.s.) + $p$. For the conventional values of the channel radii $a_1 = 4.68$, $a_2 = 4.57$ fm [8] and for our recommended value $E_{r0} = 1.54$ MeV, this contribution is proportional to $\gamma_0^2$, as in Fig. 1 of Ref. [8], reaching a value of 27 keV for $\gamma_0^2$ equal to the Wigner limit of 3.11 MeV (for $a_2 = 6.25$ fm, the upper limit on the contribution is 32 keV).

The present upper limit for the sequential two-proton decay width through the ground state of $^{11}$N is consistent with the experimental results for the two-proton decay of $^{12}$O.

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**TABLE IV.** Energies and widths (in MeV) of the 1/2$^+$ and 1/2$^-$ states in $^{11}$N from this work (Table I) and compared to previous results (values of Ref. [12] are superseded by Ref. [14], values of Refs. [6,14,16,18] are peak parameters).

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<tr>
<td>1/2$^+$</td>
<td>1.54(2)</td>
<td>0.83(3)</td>
<td>–</td>
<td>–</td>
<td>1.27$^{+0.18}_{-0.08}$</td>
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<tr>
<td>1/2$^-$</td>
<td>2.27(5)</td>
<td>1.15(25)</td>
<td>2.18(5)</td>
<td>0.44(8)</td>
<td>2.01(15)</td>
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**Theoretical results**

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$^a$If parameters in Table III of Ref. [5] are equivalent to $E_r$, $\Gamma_r$ as defined here.
the ground state of $^{12}$O [7,8], which report a width of 578(205) keV with an upper limit of 7% for $^3$He emission. Barker has already suggested that in fact the $^{12}$O experimental width may statistically be consistent with the experimental resolution of the $^{12}$O setup [7,8]. Our latest results would lend further support to this possibility and point to the need for a new higher-resolution experimental study of the two-proton decay of $^{12}$O to resolve this important issue.

V. CONCLUSIONS

A new $^{10}$C beam has been developed at the CYCLONE radioactive beam facility at Louvain-la-Neuve. The method of scattering of protons in inverse kinematics has been used to identify low-lying resonances in the proton unbound nucleus $^{11}$N. A detailed $R$-matrix analysis has yielded precise values for the resonance parameters. The value of the resonance energy for the ground state of $^{11}$N has been used to obtain an upper limit of the two-proton decay width for $^{12}$O. The result supports the present inconsistency between the current experimental two-proton decay width for $^{12}$O and a predominant sequential decay through the ground state of $^{11}$N.

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