Correlations in Amplified Four-Wave Mixing of Matter Waves

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The coherence properties of amplified matter waves generated by four-wave mixing (FWM) are studied using the Hanbury-Brown–Twiss method. We examine two limits. In the first case stimulated processes lead to the selective excitation of a pair of spatially separated modes, which we show to be second order coherent, while the second occurs when the FWM process is multimode, due to spontaneous scattering events which leads to incoherent matter waves. Amplified FWM is a promising candidate for fundamental tests of quantum mechanics where correlated modes with large occupations are required.

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The concept of second order coherence was first considered by Hanbury-Brown and Twiss (HBT), who measured intensity correlations between separate thermal light sources [1]. Although initially controversial, as it required correlations between independent photons, the concept was placed on a sound theoretical footing by Glauber, who extended the idea of coherence to arbitrary orders [2]. Second order coherence is one of the simplest measurements that shows the difference between classical and quantum particles [3]; when applied to atoms it distinguishes between Bose-Einstein condensation and thermal or chaotic sources and is a more stringent test of coherence. It has also been used to distinguish between bosonic [4,5] and fermionic sources [6,7].

These recent efforts to make correlation measurements on atoms are part of a trend to extend the well-tested techniques of quantum optics to quantum-atom optics. For example, the creation of nonclassical states of light exhibiting squeezing and entanglement is now routine, while recent advances in the field of quantum-atom optics are starting to allow the creation of nonclassical states of matter [8]. Fundamental tests of quantum mechanics that have until now only been possible with photons, such as nonlocality and the EPR paradox, are now close to being realized using massive particles [9,10].

A major goal of quantum-atom optics is to produce entangled (or at least correlated) pairs of atoms. One of the ways this can be accomplished is through atomic four-wave mixing (FWM) [11,12]. FWM in the atomic regime is achieved through the intrinsic nonlinearities in atomic interactions, in particular, atom pair collisions. The FWM process can be spontaneous or stimulated, resulting in atoms in numerous modes with low occupation or highly occupied amplified modes that result from bosonic enhancement. Bosonic stimulation is a process whereby transition rates into a particular mode are enhanced by other identical bosons already occupying that mode. Such an effect has also been used to demonstrate various processes including superradiance [13], the exponential growth of a Bose-Einstein condensate (BEC) [14], stimulated FWM [15], and the pumping of an atom laser [16].

Another difference between optical and atomic FWM is that while photons do not interact with each other, atoms emphatically do, leading to many sources of decoherence that are not present in their optical analogue. This is potentially a problem, as the coherence properties of matter waves generated via bosonic stimulation are critically important for active atom optical devices, and it is not clear that methods of producing such atomic fields result in coherence to all orders.

The coherence properties of matter waves produced in the FWM of atoms, or indeed any method involving bosonic stimulation, have only been tested to first order [13,17]. In this Letter we present the first higher order test of the coherence of amplified matter waves. We are able to access both the spontaneous regime, where we observe atom bunching due to the multimode nature of the process, and the transition to stimulated behavior where the correlation function is unity, indicating that the output modes are coherent to second order.

The concept of coherence in the sense of classical optics refers to first order or phase coherence, meaning the tendency for two field values at separated points in space or time to acquire correlated values. This is evident when the fields are superimposed and show interference fringes. The work by Glauber [2] extends the notion of coherence by defining higher order correlation functions. The nth order correlation function expresses the correlation of field values at n points in time and space, and a wave is coherent to nth order if the condition that $g^{(n)} = 1$ holds for all particle separations. Higher order coherence of matter waves can be tested using single atom detection to discern the arrival time and position of each individual atom. Using such methods the well-known HBT effect [1] has been demonstrated for both bosonic [4] and fermionic [7] atoms as well as third order coherence for a BEC [18]. The HBT effect deals with correlations in intensity fluctuations, using the normalized second order correlation function, defined spatially in terms of the intensities as

$$g^{(2)}(\delta) = \frac{\langle I(r)I(r+\delta)\rangle}{\langle I(r)\rangle\langle I(r+\delta)\rangle}, \tag{1}$$

where δ is the separation between the particles.

The HBT effect interrogates the quantum statistical properties of particles and thus is a powerful tool in determining the coherent nature of a source of matter waves. In the case of a multimode or chaotic (thermal) source of bosonic atoms, one expects $g^{(2)}(\delta)$ to give values above unity for small separations, showing the tendency for particles to arrive in bunches, thus the term "bunching."

Our experiment to test the second order coherence of matter waves generated via FWM builds on our previous observations of dynamical instabilities in a metastable helium (He*) atom laser [15]. Our experimental setup is described elsewhere [19]: briefly, we trap and evaporate He* atoms in a biplanar quadrupole Ioffe configuration (BiQUIC) magnetic trap with radial and axial trapping frequencies of $2\pi \times 565$ Hz and $2\pi \times 52$ Hz, respectively. We typically produce condensates containing $\sim 10^6$ atoms in the $m_j = +1$ state. We then generate an atom laser from the BEC by outcoupling with high power radio frequency (rf) radiation (Rabi frequency $\Omega \sim 500$ Hz) for 10 ms. To enable consistent FWM production it is crucial to ensure stable rf outcoupling from the magnetic trap, which requires active cancellation of background ac magnetic fields [20]. Outcoupled $m_j = 0$ atoms fall under gravity and are detected with a delay-line detector located 848 mm below trap center; see Fig. 1. This provides us with a position and arrival time for each individual atom. Characterization of our detector is reported in [21]. Atoms in substate $m_j = -1$ are also produced in the outcoupling, especially at high rf powers. They are, however, accelerated away from the detector by the trapping field.

Tuning the rf to the high density region of the BEC, in the strong outcoupling limit, results in an atom laser profile which contains four peaks; see Fig. 1. The peaks arise from amplified atomic FWM modes, as predicted by a full three-dimensional numerical simulation and observed in our previous experiments [15].

The FWM in our system is generated through an unusual process that occurs due to a mismatch between the s-wave scattering lengths of different magnetic substates of the BEC. The scattering length for $m_j = 1{:}m_j = 1$ collisions is $a_{11} = 7.51$ nm [22], which also applies to 0:1 collisions. This value can be used to constrain the uncertainty in the $^5\Sigma_g^+$ potential and obtain a scattering length of $a_{00} = 5.56$ nm for 0:0 collisions from Ref. [23], a value which is 26% smaller. As the nonlinear mean field energy density between two states is given by $4\pi\hbar^2 a_{ij}|\psi_i|^2|\psi_j|^2/m$, where $|\psi_i|^2$ is the local density in the state $m_j = i$, we have the intriguing possibility that an $m_j = 0$ and an $m_j = 1$ atom, initially both *stationary*, can scatter off each other, acquiring an equal and opposite (but nonzero) velocity. These moving atoms establish a density grating in the background BEC, which lowers the overall nonlinear mean field energy [15] and allows the seeding of momentum modes without actively colliding two moving condensates.

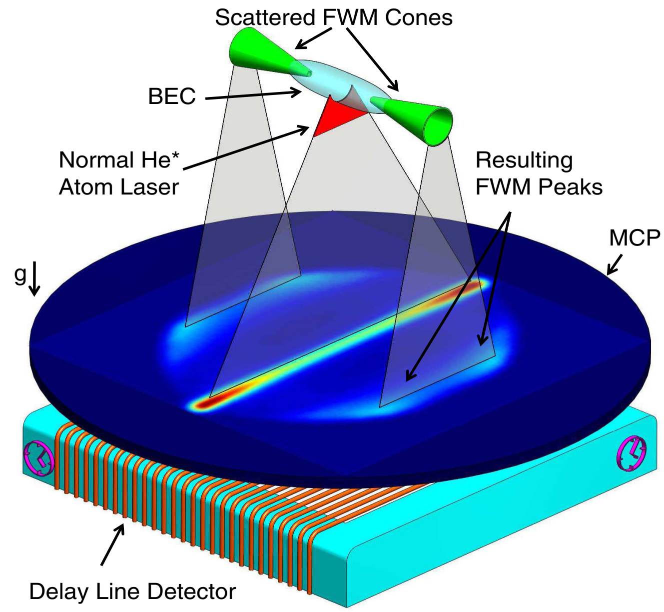

FIG. 1 (color online). Experimental schematic showing the BEC, the atom laser, and the cones that result from the FWM process (not to scale). The resulting image on our multichannel plate (MCP) and delay line detector shows peaks due to the FWM cones.

The fact that the creation of nonzero momentum modes in a stationary BEC is energetically allowed does not necessarily mean that any of these modes will become significantly populated through stimulated Bose enhancement. To determine whether this occurs we require an analysis of the stability of the BEC beyond the mean field solution, i.e., a Bogoliubov approach. A full analysis can be found in [24], but we provide a brief outline here.

The problem is to determine the response of the BEC to small fluctuations about the mean field. Typically the BEC is stable to such fluctuations, and their energy spectrum determines the phase and group velocities of the excitations; in a standard BEC one obtains the usual Bogoliubov quadratic dispersion relation [25]. However, if the system is a multilevel BEC with mismatched scattering lengths and there is a strong coupling between the levels, this is no longer the case, and the equations of motion for the fluctuation operators can possess imaginary eigenvalues at specific momentum values. In this case the BEC is said to be dynamically unstable, and there is exponential growth of the population at those momenta.

Finally, if the Thomas-Fermi radius of the BEC is less than the de Broglie wavelength of the momentum at which amplification occurs, the local density approximation is no longer satisfied, and no amplification (and hence no FWM) occurs. This means that in a high-aspect ratio BEC only momentum modes predominantly in the direction of the long axis of the BEC will be amplified, resulting in matter wave cones being emitted from the BEC, resulting in four peaks when observed on a horizontal plane below (see Fig. 1). The higher the aspect ratio, the narrower the cones; the limiting behavior is two low-divergence, oppositely directed, matter wave beams.

Because the density grating which enables the amplified FWM takes time to build up (~ 3 ms [15]), there will be two distinct FWM processes within a single experimental sequence, each with different coherence properties. Initially, while the grating is forming, spontaneous FWM dominates, resulting in atoms scattering into a large number of weakly populated modes yielding an s-wave shell [see Fig. 2(a)]. While each individual scattering event results in two atoms with opposite momenta that are correlated with each other, the multimode nature of the process means the overall atomic field of the resulting halo is incoherent. However, once the grating is established, the specific resonant momentum modes become amplified,

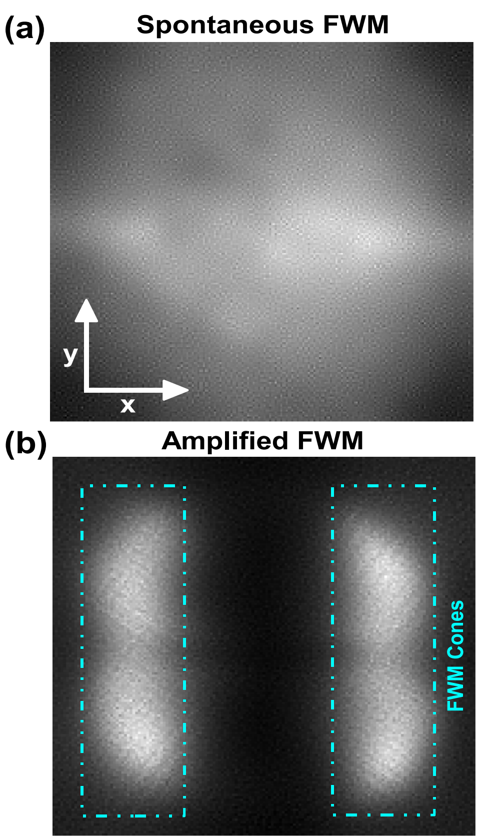

FIG. 2 (color online). Typical intensity plots (arb. units) for (a) s-wave scattering shell and (b) the amplified FWM waves as seen on the detector. The tight trapping direction is along the y axis, while the x axis corresponds to the weak trapping direction. Each image is 3 cm $\times$ 3 cm.

leading to a few- or single-mode system [24]. These modes form the FWM cones that result in the peaks we see on our detector [see Fig. 2(b)], which are predicted to be coherent. In order to compare the two different regimes, we select temporal windows which divide the atoms into the spontaneous or amplified cases, as well as spatial windows to remove the unscattered atoms which form an atom laser. The spontaneous scattering is analyzed over a 3 ms window, while the amplified FWM is from a 10 ms window, with a 4 ms period in between the two limiting cases. While it would be interesting to probe the evolution of the correlation function on shorter time scales, pulse spreading (~ 10 ms) due to mean field effects currently prevents such an investigation.

We calculate the second order correlation function for the s-wave scattering shell and the amplified FWM modes as detailed in [21]. For each case the analysis was done using bins with spatial widths of 200 μm in both dimensions, and 50 μs in time. Since atoms at the detector move at $\sim$4 ms^{-1}, in the vertical plane, this corresponds to an equivalent spatial bin distance for all dimensions. The analysis of the second order correlation function of the FWM signals is carried out in the spatial regions bounded by the boxes shown in Fig. 2 along the y axis [direction indicated in Fig. 2(a)]. The normalized correlation function for the predominantly s-wave scattering [Fig. 3(a)] exhibits bunching as predicted for multimode sources [11], with a bunching amplitude of 1.076(8) and a correlation length of 860(90) μm. The amplified FWM cones, on the other hand, have $g^{(2)} = 1.000(7)$ for all separations indicating clearly that the amplified modes are coherent to second order. The stated error values are from all sources, including shot noise, normalization uncertainties, and technical noise.

To estimate the theoretical correlation length for the spontaneously scattered case, we follow the approach taken in [11] using the mean velocity of the BEC $v_{\rm rms} = 2\hbar/mR$, where R is the Thomas-Fermi radius. Using our trapping frequency, chemical potential ($\mu \sim 8$ kHz), and fall time we obtain a correlation length of $\sim$1.2 mm. This is remarkably close given factors such as the outcoupling region being only a small part of the BEC, the quasicontinuous nature of the outcoupling, and interactions with trapped atoms.

The observation that amplification of the initial matter wave maintains the coherence of the source to second order is not a given, as atomic interactions can lead to many sources of decoherence that are present during the collisional amplification process and subsequent separation of the matter wave from the BEC. The major source of decoherence is due to collisions between the amplified wave and the trapped source (BEC) [26]. Furthermore, phase fluctuations are written onto the amplified matter wave and the nonuniform mean field induces spatially dependent phase shifts [27].

The high visibility interference fringes as reported in [13,17] showed that amplified matter waves exhibit

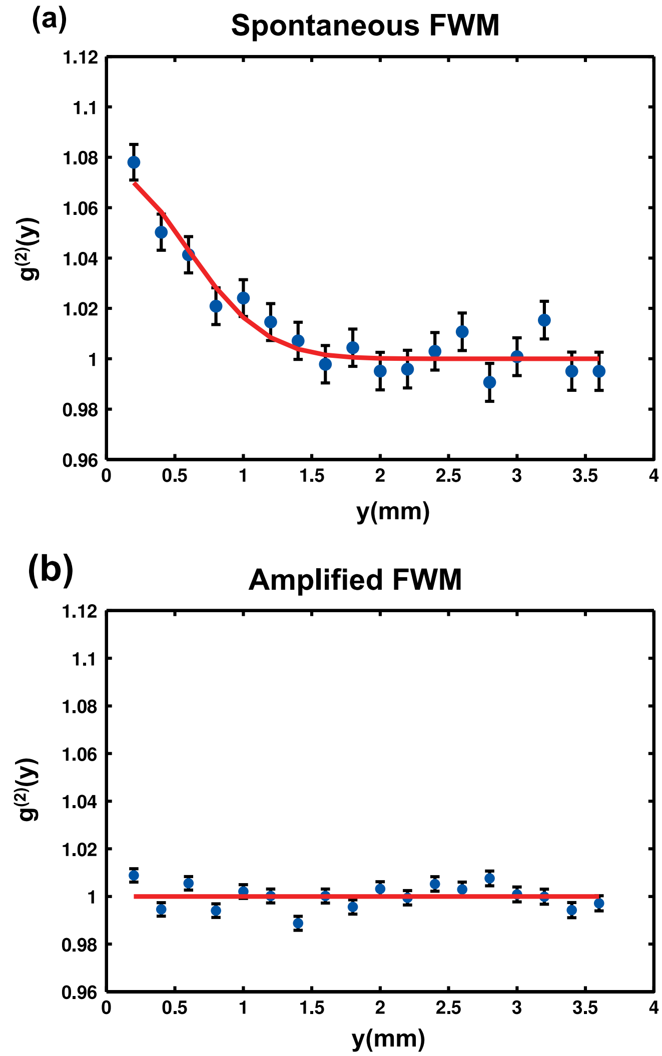

FIG. 3 (color online). Normalized second order correlation functions for (a) spontaneous FWM events, which form an *s*-wave scattering shell, and (b) the amplified FWM cones. Error bars indicate the contribution due to shot noise. The spontaneous case shows clear bunching, whereas the amplified modes show a correlation function of unity.

long-range phase coherence. In our experiment, the absence of bunching demonstrates second order coherence of matter waves amplified by bosonic stimulation, an important condition for complete coherence as defined in [2].

It has been predicted that the process described here should produce relative number squeezing between oppositely directed FWM modes, as demonstrated by Jaskula *et al.* [28] for spontaneous FWM in a similar system. Were such squeezing to be present in our case, the coherence of our single-mode process would greatly increase the practical applications, i.e., in squeezed atom laser interferometry [29]. The modes may also be EPR entangled, but whether this type of entanglement survives the outcoupling process is still an open question.

In conclusion we have demonstrated the second order coherence of matter wave beams arising from amplified FWM compared to the spontaneous case which exhibits bunching. Our experiment confirms that by applying rf radiation with the correct parameters to a He* BEC, a coherent FWM process arises, resulting in amplified single-mode matter waves. In future experiments we hope to demonstrate relative number squeezing between opposite FWM waves. If such squeezing is confirmed for coherently amplified waves, then our method would be a simple way to create pairs of squeezed atom lasers, which should offer subshot noise performance in future atom interferometers [29]. It may also allow the possibility of useful EPR-type entanglement between the modes [24].

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