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Backward Tamm states in left-handed metamaterials

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The authors study the electromagnetic surface waves localized at an interface separating a one-dimensional photonic crystal and left-handed metamaterial, the so-called surface Tamm states. They demonstrate that the metamaterial allows for a flexible control of the dispersion properties of surface states and can support the Tamm states with a backward energy flow and a vortexlike structure. © *2006 American Institute of Physics*. [DOI: 10.1063/1.2352794]

The study of unusual properties of *left-handed metamaterials* is a rapidly developing field of physics. Left-handed metamaterial (LHM) is characterized by simultaneously negative effective dielectric permittivity and negative effective magnetic permeability, which gives rise to a variety of unusual properties of electromagnetic waves. Such materials have first been suggested theoretically by Veselago[1] almost 40 years ago, but they have been realized experimentally only a few years ago.[2] Later, it was shown that they possess many extraordinary waveguiding properties.[3–6] In particular, an interface between conventional dielectric and LHM is able to support either TE- or TM-polarized guided modes,[5] which is impossible for an interface between conventional dielectrics. In this letter we demonstrate another example of unusual properties of LHM and study electromagnetic surface waves guided by an interface between LHM and a one-dimensional photonic crystal, the so-called *surface Tamm states*.[7] We demonstrate that the presence of a metamaterial allows for a flexible control of the dispersion properties of surface states, and the interface can support the Tamm states with a backward energy flow and a vortexlike structure.

Surface modes are a special type of waves localized at an interface between two different media. In periodic systems, staggered modes localized at surfaces are known as Tamm states,[7–9] first found in solid-state physics as localized electronic states at the edge of a truncated periodic potential. Surface states have been studied in many different fields of physics, including optics,[10,11] where such waves are confined to an interface between periodic and homogeneous dielectric media, as well as nonlinear dynamics of discrete chains.[12] In optics, the periodic structures have to be manufactured artificially in order to manipulate dispersion properties of light in a similar way as the properties of electrons are controlled in crystals. Such periodic dielectric structures are known as photonic crystals. An analogy between solid-state physics and optics suggests that surface electromagnetic waves should exist at the interfaces of photonic crystals, and indeed they were predicted theoretically[10,11] and observed experimentally.[13] Such Tamm states can be very important for applications of photonic crystals, as they allow for the enhanced coupling of the electromagnetic waves to and from the photonic crystal waveguides.[14,15]

In this letter, we study surface electromagnetic waves, or surface Tamm states, guided by an interface separating homogeneous LHM and one-dimensional photonic crystal. We assume that the terminating layer (or a cap layer) of the periodic structure has the width different from the width of other layers of the structure. We study the effect of the width of this termination layer on surface states and explore a possibility to control the dispersion properties of surface waves by adjusting termination layer thickness. We find *unusual* types of surface Tamm states at the interface with the metamaterial which have a backward energy flow and a vortexlike structure. We also compare our results with the case when the LHM medium is replaced by a conventional dielectric, which we refer to as right-handed material (RHM). The surface states in these two cases we call left- and right-handed Tamm states, respectively.

Geometry of our problem is sketched in Fig. 1. We consider the propagation of TE-polarized waves described by one component of the electric field $E=E_y$,[16] and governed by a scalar Helmholtz-type equation. We look for stationary solutions propagating along the interface with the characteristic dependence $\sim\exp[-i\omega(t-\beta x/c)]$, where ω is the angular frequency, β is the normalized wave number component along the interface, and c is the speed of light, and present this equation in the form

$$\left[\frac{d^2}{dz^2}-k_x^2+\frac{\omega^2}{c^2}\varepsilon(z)\mu(z)-\frac{1}{\mu(z)}\frac{d\mu}{dz}\frac{d}{dz}\right]E=0, \qquad (1)$$

where $k_x^2=\omega^2\beta^2/c^2$, and both $\epsilon(z)$ and $\mu(z)$ characterize the transverse structure of the media. Surface modes correspond to localized solutions with the field E decaying from the interface in both the directions. In a left-side homogeneous

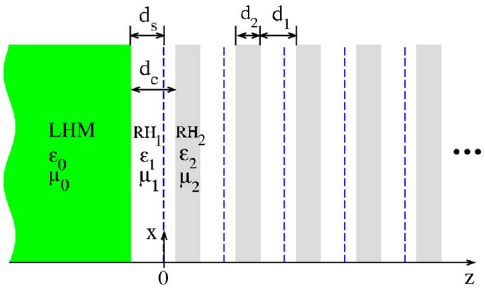

FIG. 1. (Color online) Geometry of the problem. In our calculations we take the following values: $d_1=1$ cm, $d_2=1.65$ cm, $\varepsilon_1=4$, $\mu_1=1$, $\varepsilon_2=2.25$, $\mu_2=1$, $\varepsilon_0=-1$, and $\mu_0=-1$.

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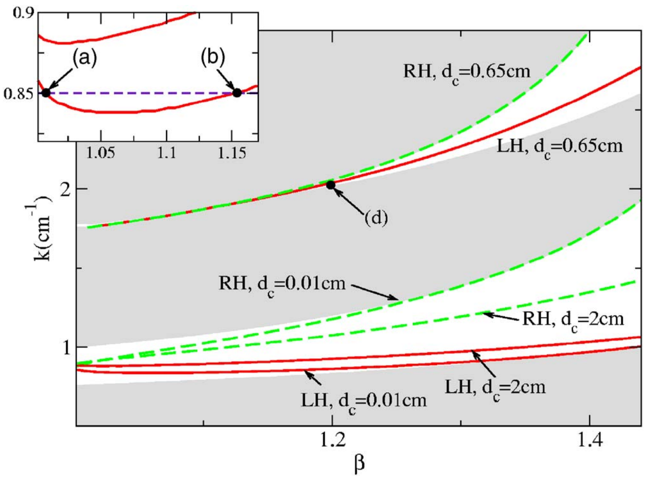

FIG. 2. (Color online) Dispersion properties of the Tamm states in the first and second spectral gaps. Shaded: Bands of the one-dimensional photonic crystal. Solid: Dispersion of the Tamm states with LHM. Dashed: Dispersion of the surface waves in the problem where the metamaterial is replaced by vacuum. Corresponding values of the cap layer thickness are indicated next to the curves. Inset shows a blowup region of small β. Points (a), (b), and (d) correspond to the mode profiles presented in Figs. 3(a), 3(b), and 3(d).

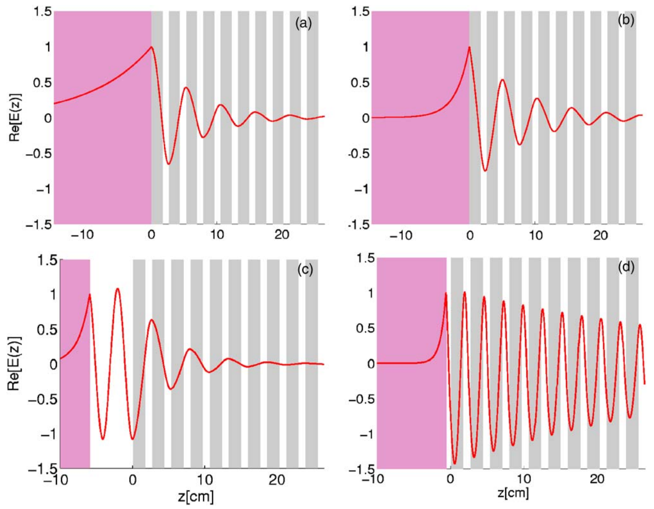

FIG. 3. (Color online) Examples of the left-handed surface Tamm states. (a) Backward LHM mode: d_c=0.01 cm, k=0.85 cm^{-1}, and β=1.008. (b) Forward LHM mode: d_c=0.01 cm, k=0.85 cm^{-1}, and β=1.153: (c) Guided mode with a thick cap layer: d_c=6 cm, k=0.9561 cm^{-1}, and β=1.2. (d) Surface mode in the second band gap: d_c=0.65 cm, k=2.0386 cm^{-1}, and β=1.2. Modes (a), (b), and (d) correspond to the points in Fig. 2.

medium ($z<-d_s$, see Fig. 1), the fields are decaying provided $\beta>\varepsilon_0\mu_0$. In the right-side periodic structure, the waves are the Bloch modes,

$$E(z)=\Psi(z)\exp(iK_B z), \tag{2}$$

where K_b is the Bloch wave number, and $\Psi(z)$ is the Bloch function which is periodic with the period of the photonic structure (see details, e.g., in Ref. 16). In the periodic structure the waves will be decaying provided K_b is complex; and this condition defines the spectral gaps of an infinite photonic crystal. For the calculation of the Bloch modes, we use the well-known transfer matrix method.[17]

To find the Tamm states, we take solutions of Eq. (1) in a homogeneous medium and the Bloch modes in the periodic structure and satisfy the conditions of continuity of the tangential components of the electric and magnetic fields at the interface between homogeneous medium and periodic structure.[18] We summarize the dispersion properties of the Tamm states in the first and second spectral gaps on the plane of the free-space wave number $k=\omega/c$ versus the propagation constant β (see Fig. 2) for different values of the cap layer thickness d_c. For comparison, we also plot the dispersion of the corresponding Tamm states in the structure, where the homogeneous medium is replaced by vacuum (dashed).

As mentioned above, the Tamm states exist in the gaps of the photonic band gap spectrum (unshaded regions in Fig. 2). For a thin cap layer, d_c=0.01 cm, the left-handed Tamm state approaches the lower edge of the first (lower) band gap, while this right-handed Tamm state approaches the top edge. Another important difference between the two cases is that for the LHM surface modes the slope of the dispersion curve becomes negative for small β, and it remains positive for larger longitudinal wave numbers (see the inset in Fig. 2), while for the conventional dielectric media, the dispersion curves are always with a positive slope. The slope of the dispersion curve determines the corresponding group velocity of the mode. The extended control over the group velocity in the case of LHM band gap structure is possible due to the backward energy flow in metamaterials. Similar effects have been already predicted for other types of the waveguiding structures.[5,6] Similarly, the left-handed Tamm surface wave has a *vortexlike* energy flow pattern.

As a result of different slopes of the dispersion curve of the left-handed Tamm states, we observe the mode degeneracy, i.e., for the same frequency ω (or wave number k), there exist two modes with different values of β. The mode with lower β has a negative group velocity (with respect to the propagation wave vector), while the other mode has a positive group velocity. Such modes are termed as *backward* and *forward*, respectively. In the forward wave, the direction of the *total energy flow* coincides with the propagation direction, while in the backward wave the energy flow is backward with respect to the wave vector. Physically, this difference can be explained by looking at the transverse structure of these two modes. In Figs. 3(a) and 3(b) we plot the profiles of the two modes having the same frequency $k=\omega/c$ $=0.85$ cm^{-1}, with different longitudinal wave numbers β. Corresponding points are shown in the inset in Fig. 2. For mode (a), the energy flow in the metamaterial exceeds that in the periodic structure (slow decay of the field for $z<-d_s$ and fast decay into the periodic structure), thus the total energy flow is backward. For mode (b) we have the opposite case, and the mode is forward. In the right-handed Tamm state

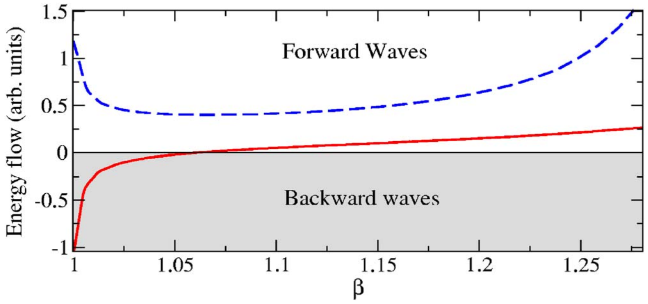

FIG. 4. (Color online) Total energy flow in RH (dashed) and LH (solid) Tamm modes vs β. Cap layer thickness is d_c=0.01 cm.

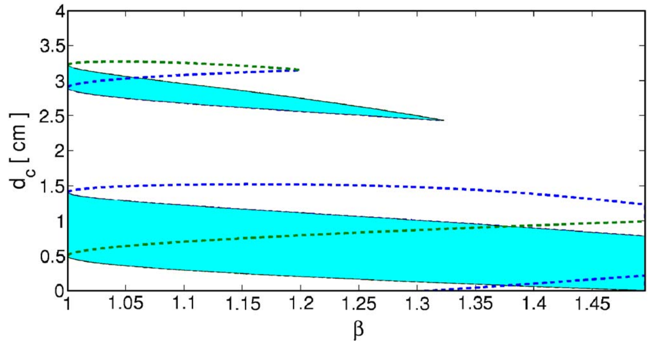

FIG. 5. (Color online) Existence regions of the surface Tamm modes; the modes do not exist in the shaded regions. Dashed curves mark the corresponding regions for the right-handed Tamm states.

geometry, the energy flow in all parts of the wave is directed along the wave number and the localized surface waves are always forward. To demonstrate this, in Fig. 4 we plot the total energy flow in the modes as a function of the wave number β. These results confirm our discussion based on the analysis of the dispersion characteristics.

Increasing the thickness of the cap layer d_c will push the dispersion of both right- and left-handed modes inside the gap (see the curves for $d_c=0.65$ cm in Fig. 2), thus providing better localization of the modes. In the second gap, for the chosen parameters the LHM mode exists deeper in the gap than the right-handed mode, allowing for a better wave localization. An example of the second-gap-mode profile is shown in Fig. 3(d). One can see that the second band modes are generally weaker localized at the interface than the modes from the first band gap.

Finally, in Fig. 5 we plot the existence regions for the surface Tamm modes on the parameter plane (d_c,β). Shaded is the area where the surface modes *do not exist*. Increasing the cap layer thickness d_c, we effectively obtain a dielectric waveguide, one cladding of which is a homogeneous medium, while the other one is a photonic crystal. In such a case a typical mode is shown in Fig. 3(c). The contours of the corresponding nonexistence regions for right-handed Tamm states are also shown in Fig. 5. The regions have qualitatively different shapes. While the lower nonexistence region moves down with increase of β preserving its width, the corresponding right-handed region shifts upward, decreasing significantly in width.

In conclusion, we have studied electromagnetic surface waves guided by an interface between a left-handed metamaterial and a one-dimensional photonic crystal. We have shown that in the presence of a left-handed material the surface Tamm waves can be either forward or backward while for conventional structures the Tamm states are always forward. We have compared the properties of the backward Tamm states with the case when left-handed material is replaced by conventional dielectric and analyzed the existence regions for both right- and left-handed Tamm states. We believe that our results will be useful for a deeper understanding of the properties of surface waves in plasmonic and metamaterial systems.

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