Investigation and comparison of multistate and two-state atom laser-output couplers

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We investigate the spatial structure and temporal dynamics created in a Bose-Einstein condensate by radio-frequency atom laser-output couplers using a one-dimensional mean-field model. We compare the behavior of a “pure” two-state atom laser to the multilevel systems demonstrated in laboratories. In particular, we investigate the peak homogeneous output flux, classical fluctuations in the beam, and the onset of a bound state which shuts down the atom laser output.

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I. INTRODUCTION

The analogy between atom lasers and optical lasers is strong [1]. Both optical and atom lasers create a coherent output beam of bosons that are photons in the case of an optical laser and de Broglie matter waves in the case of an atom laser. The lasing mode in an optical laser is pumped through a nonthermal equilibrium process and is not the lowest energy mode of the cavity but, rather, is a highly excited mode many wavelengths long. For atoms, the lasing mode is populated through Bose-Einstein condensation and is the ground state of the trap. In an optical laser, a beam is out-coupled using a grating or a partially reflecting mirror. Most atom laser beams studied to date have been out-coupled either continuously (long pulse) or pulsed using rf radiation to transfer atoms from a trapped magnetic substate to a state that does not interact with the trapping field [2–4]. The atoms fall away from the trap under gravity producing a coherent matter wave [5–8].

Optical lasers have found broad applications in precision measurements to address questions both fundamental and applied in nature. In many cases, we expect to be able to perform such experiments more effectively and to higher precision with atom interferometry [9,10]. Measurements made with the low density, highly coherent beam from an atom laser may not be limited in precision by the mean-field interaction that plagues interferometric measurements made with more dense atomic sources such as full Bose-Einstein condensates [11,12].

We have recently found that the rf output coupler has a number of properties which hinder the production of a high flux, shot-noise limited atom laser [13,14]. We have found that a tradeoff must be made between output flux and classical (or dynamical) fluctuations in the output beam. We have measured that the peak homogeneous flux into the atom laser beam is significantly below the flux that can be provided by the finite reservoir of Bose-Einstein condensate (BEC) atoms in a typical experiment. This homogeneous flux limit is imposed by the interaction between the inherent multiple internal Zeeman states of the magnetically confined atoms. Furthermore, we have also found that a previously predicted effect known as the bound state of an atom laser [15] effectually shuts off the rf output coupler and hence the atom laser beam.

Recently, two-state atom laser systems have been produced experimentally [4,16] with the aim of avoiding classical fluctuations existing in more complicated multistate systems. Le Coq et al. produce a $^{87}$Rb BEC in the $F=1, m_F=-1$ state in a harmonic trap with a large bias magnetic field (40 G). An atom laser is created by applying an rf coupling resonant with a transition from the $m_F=-1$ state to the $m_F=0$ state. Transitions to the antitrapped $m_F=1$ state are suppressed by the second order Zeeman shift, creating an effective two-state atom laser. Öttl et al. also create a $^{87}$Rb BEC in the $F=1, m_F=-1$ state. The authors then use a microwave transition at 6.8 GHz to resonantly couple atoms to the $F=2, m_F=0$ state. The magnetic trap bias field splits the Zeeman states in the $F=2$ manifold by approximately 1 MHz, once again leading to a two-state system.

In this paper we utilize a 1D numerical model to investigate the spatial dynamics of an output coupler for five-, three-, and two-level atom laser systems. The five- and three-state systems correspond to the experimentally relevant Zeeman levels of the $F=2$ and $F=1$ ground states of $^{87}$Rb and the two-state simulations to the recent experiments of Le Coq et al. and Öttl et al. The question we answer is whether the significant amount of experimental effort to make a two-state system has an actual influence on the quality of the atom laser. We analyze each of these systems with respect to experimentally important properties. In particular, we investigate the peak homogeneous output flux, classical fluctuations in the beam, and the onset of a bound state which shuts down the atom laser output.

We find that, in the weak coupling regime, the three-state and pure two-state systems are indistinguishable with respect to classical fluctuations and flux. We conclude that the added experimental complications of producing a truly pure two-state system do not represent a significant benefit.

II. MODEL

An rf output coupler uses the resonance between nondegenerate Zeeman states in the small bias field at the mini-
mum of the magnetic trap. Applying a monochromatic rf magnetic field resonant with this splitting coherently couples atoms between the relevant Zeeman states. The magnetically trapped $F=2,m_F=\pm 2$ atoms can then be “shunted” through the $F=2,m_F=1$ state to the $F=2,m_F=0$ state in which they no longer interact with the magnetic potential. The presence of the trapping magnetic field also introduces a spatial reso-
nance associated with a given frequency allowing the center of the resonance to be tuned within and around the condensa-
tate. The resonance condition is satisfied on the surface of an ellipsoid centered around the minimum of the magnetic field. Gravity introduces an asymmetry such that the minimum of the trapping potential given by $\frac{1}{2}m \omega_z^2 \rho^2$ is shifted down vertically by $G_{\text{shift}} = g/\omega_0^2$ from the center of the magnetic field minimum. Here $m$ is the atomic mass, $g$ the acceleration due to gravity, and $\omega_0$ the radial trapping frequency of the $F=2,m_F=2$ state. This asymmetry produces a “preferred” direction of the coupling process and the atoms fall out of the trapping region.

### A. Time dependent equations

Ballagh et al. [17] introduced the Gross-Pitaevskii (GP) equation as an effective tool for investigating this type of atom laser within the semiclassical mean-field approximation. A number of groups have found good agreement between theory and experiment [18–20] using mean-field models of the atom laser. Numerical descriptions of an atom laser within the mean-field framework are complicated by the large velocities that atoms reach when falling in a gravitational potential. The resultant small de Broglie wavelengths require very fine temporal and spatial numerical grids in order to accurately follow the dynamics. Additionally, to run simulations that reflect experiments on a time scale longer than a few milliseconds, an apodising mechanism must be introduced to absorb the beam and hence avoid breakdown of the numerical techniques used. These factors make simu-
lating and understanding atom laser dynamics a complicated proposition. In order to simplify the numerics, a true 3D system can be transformed to lower dimensions. The dimensionality reduction can be performed nonrigorously by writing an equivalent equation for the system in the dimension(s) of interest [18]. The $F=2$ GP model [13] of the atom laser in one dimension (the coordinate over which gravity acts) is then given by

$$i \dot{\phi}_2 = (L + z^2 + Gz + 2\Delta) \phi_2 + 2\Omega \phi_1,$$

$$i \dot{\phi}_1 = \left( L + \frac{1}{2} z^2 + Gz - \Delta \right) \phi_1 + 2\Omega \phi_2 + \sqrt{6} \Omega \phi_0,$$

$$i \dot{\phi}_0 = (L + Gz) \phi_0 + \sqrt{6} \Omega \phi_1 + \sqrt{6} \Omega \phi_{-1},$$

$$i \dot{\phi}_{-1} = \left( L - \frac{1}{2} z^2 + Gz + \Delta \right) \phi_{-1} + 2\Omega \phi_{-2} + \sqrt{6} \Omega \phi_0,$$

where $\phi_i$ is the GP function for the $i$th Zeeman state and $L = -\frac{1}{2} (\partial^2 / \partial z^2) + U(\sum_{i=2}^3 \phi_i^2)$. Here $\Delta$ and $\Omega$ are respectively the detuning of the rf field from the $B_0$ resonance, and the Rabi frequency, measured in units of the radial trapping frequency $\omega_0$ (for the $F=2$, $m_F=1$ state), $U$ is the interaction coefficient, and $G = (mg/\hbar \omega_0)(\hbar/m \omega_0)^{1/2}$ gravity. The wave functions, time, spatial coordinates, and interaction strengths are measured in the units of $(\hbar/m \omega_0)^{-1/4}$, $\omega_0^{-1}$, and $(\hbar/m \omega_0)^{-1/2}$, respectively. The nonlinear interaction strength is derived by requiring that the one-dimensional (1D) Thomas-Fermi chemical potential be equivalent to the 3D case. There are no free parameters in this model; we use $U=6.6 \times 10^{-4}$, $G=9.24$, $\Omega=0-14$, and $\Delta=10.7$. Although all the numerics are executed in dimensionless units, data are presented in dimensional units.

Fortuitously, the $g_F$ factors for the $F=2$ and $F=1$ ground states of $^{87}\text{Rb}$ are the same except for a change of sign. The three-state and two-state atom laser systems are then simply subsets of the five-state equations presented above, with the modification that the prefactor $\sqrt{6}$ in the coupling terms between the $m_F=\pm 1,0,1$ states becomes $\sqrt{2}$.

Our simulations were performed using Matlab, a commercially available and widely used software packing, using a Fourier based, symmetric split step algorithm. For this work we use a 2048 point spatial grid from $-40$ to $40$ with the equilibrium position of the condensate at $20$ and a temporal resolution of $10^{-4}$. Apodising boundaries for each state have an exponential form and are positioned in order that no spatial aliasing occurs.

In addition to solving the time-dependent GP equations for the atom laser we also find the time-independent solutions to provide accurate initial conditions for our simulations. The time-independent GP equation can be found from Eq. (1) using the substitution $\phi_i(r,t)=\varphi(r)e^{i\mu t}$. We use a relaxation technique [21] which, if required, allows us to find excited stationary states as well as the ground state solution. For the five-state system we use as our initial condition the solution to the following time-independent equation:

$$\mu_2 = \left( -\frac{1}{2} \frac{\partial^2}{\partial z^2} + U_2 |\phi_2|^2 + z^2 \right) \phi_2.$$  

For the two- and three-state systems we solve

$$\mu_1 = \left( -\frac{1}{2} \frac{\partial^2}{\partial z^2} + U_1 |\phi_{-1}|^2 + \frac{z^2}{2} \right) \phi_{-1}.$$  

In these equations the 1D interaction strengths are calculated by requiring that the one- and three-dimensional chemical potentials are equal.

### III. RESULTS

In this section we present the general results for each of the five-, three-, and two-state systems. Unless otherwise stated, for each system studied the rf coupling resonance is arranged to be at the centre of the trapped BEC by selecting an appropriate value for the detuning.

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A. Five-state system

We first analyze the five-state system. In Fig. 1 we show the population dynamics for strong and weak coupling. Although we are using an apodising boundary in the numerical simulations we keep track of the total population in each state by calculating at each time step the one-dimensional flux passing through a point (we like to think of it as a detector) on the numerical grid, significantly below the trapped condensate and above the apodising boundary. Hence the numerical grid is effectively divided into two sections: Above and below this “detector.” At any time during the simulation we can determine the total number of atoms in a particular state by summing the number of atoms on the grid above the detector with the total number of atoms that have passed through the detector. Apart from giving results that are more intuitively obvious, this method allows us to monitor the normalization of the numerics. A number of theoretical works have suggested that the atom laser may have a “bound” eigenstate [15,22,23], based purely on the existence of coupling between a single trap mode and a continuum of untrapped states. Furthermore, in the context of producing a two-dimensional BEC, it has been shown recently that trapping of all $m_F$ states is a natural consequence of combining rf coupling with a dc magnetic trap [24,25]. This trapping can be understood by considering the “dressed-state” basis in which the rf coupling and dc potentials seen by the atoms are diagonalized. In this basis the dressed eigenstates are linear combinations of the bare Zeeman states, trapped in effective diagonalized. In this basis the dressed eigenstates are linear which the rf coupling and dc potentials seen by the atoms are operative. Some small modulation introduced by the output-coupling mechanism of the $m_F=1$ state has a different equilibrium position to the $m_F=2$ state (the gravitational sag is different for each state) the $m_F=1$ atoms start to oscillate in the magnetic trap. The dynamics of such an oscillation is shown in Fig. 2. This phenomenon is independent of the output-coupling strength and indicates that even at low flux a five-state atom laser system will be modulated by classical noise. Density fluctuations in the output beam are shown in Fig. 3 for different Rabi frequencies in the weak coupling regime. We note here that the spatial dynamics imposed on the atom laser beam by the mechanism of the $m_F=1$ sloshing in the trap are an independent noise mechanism compared to the back-coupling dynamics investigated previously [13].

B. Comparison of the theoretical model with respect to previous experiments

Since the stated goal of this paper is to guide future atom laser experiments, it is important to validate the 1D mean-
field model used here. We confirm the accuracy of our work by comparing numerical simulations of the five-state atom laser with experimental data for an $F=2$, $m_F=2$ system obtained previously [14]. We find excellent qualitative agreement between theory and experiment, with no free parameters in our model. The five-state system is the most computationally and physically complex of the five-, three-, and two-state atom lasers and hence we have confidence that our models for the three- and two-state systems are also qualitatively accurate.

The experimental system we will compare our five-state system to is described in detail in our previous work [14]. $^{87}$Rb condensates of around $10^5$ atoms are produced in a highly stabilized magnetic trap which enables precise, repeatable, and highly calibrated rf output coupling of the condensate. The radial trapping frequency of the system is $\nu_r=260$ Hz and the axial trapping frequency is $\nu_z=20$ Hz at a bias field of $B_0=0.25$ G.

In Fig. 4, the upper plot (a) shows a measurement of the total atomic population in the $m_F=2,1$ states remaining in the magnetic trap after 100 ms of output coupling as a function of angular Rabi frequency. The prediction of the 1D model (solid line) qualitatively matches the behavior of the experiment (open circles). The numerical simulation has no free parameters. The quantitative discrepancy between theory and experiment can be explained, in most part, by the reduced dimensionality of the simulation. We have no reason to expect that a full 3D model would not capture the dynamics of our experiment and improve the quantitative agreement of the numerics with the measurements, but such simulations are beyond the scope of the present work.

In Figs 4(b)–4(e), we compare theory and experiment for a short period of continuous output coupling, once again with no free parameters in our model. The dashed lines represent experimental optical depth plots, integrated across the

**FIG. 3.** Density fluctuations in the $m_F=0$ state of a five-state system at a single point in the beam as a function of time. The fluctuations due to the sloshing of the $m_F=1$ are increasingly severe as the coupling strength is increased. Parameters are $U=6.6 \times 10^{-3}$, $G=9.24$, and $\Delta=10.7$.

**FIG. 4.** (Color online) Comparison of numerical calculations with previous experimental results (a) the total atomic population remaining in the magnetic trap after 100 ms of output coupling as a function of angular Rabi frequency for both our numerical model (solid line) and some of our previous experimental measurements (open circles). One can see the excellent qualitative agreement between both curves, (b)–(e) compare theoretical (solid lines) and experimental (dashed lines) results showing the spatial structure of an atom laser for a short time of continuous output coupling and for different Rabi frequencies. Note that all numerical simulations have no free parameters. The detailed description of the figure is given in the text.
atom laser beam profile in the direction perpendicular to gravity, showing the spatial structure of an atom laser for different Rabi frequencies. In this experiment (Fig. 1 of Ref. [14]), a short burst of output coupling lasted for 3 ms, the system was then left to evolve for a further 4 ms before the trap was switched off, and 2 ms later the image was acquired, using standard absorption imaging. The theoretical results (solid lines) were obtained for 2.6 ms of output coupling followed by transformation of the numerical data to account for 6 ms of free fall. Because of the numerical issues discussed earlier, we could not simulate the full 3 ms of output coupling, while keeping all outcoupled atoms on the numerical grid. However, the numerical results presented here capture well the spatial structure of the atom laser beam.

As the total time over which atoms are accelerated is different between theory (8.6 ms) and experiment (9 ms), we plotted the corresponding results on slightly different horizontal scales in order to compare them. Indeed, the axis for numerics is displayed on the top of each graph, whereas the bottom scale is displayed for experimental results. The two curves have been superimposed by a linear adjustment of the horizontal axis. Moreover, although the theoretical data is given in arbitrary units, it was vertically scaled in order to satisfy the conservation of the number of atoms. The main quantitative discrepancy from our comparisons occurs in the width of some peaks of density. We can explain this result by the fact that the natural broadening due to dispersion was not taken into account in our simulations. In this approximation, the width of the peaks obtained from the numerics is expected to be narrower than the experimental results. However, acceleration of the output-coupled atoms also leads to a broadening which becomes more and more important as they keep falling under gravity. For the atoms to the right of the pictures, which have fallen further away, dispersion seems to have a negligible influence on the broadening compared to acceleration under gravity. In this case our simulations are able to model very well the spatial structure of the atom laser beam. On the other hand, the atoms to the left of the pictures have only experienced about 3 ms of free fall and one can see dispersion cannot be neglected.

C. Three- and two-state systems

The three-level system offers the possibility of a cleaner output than the five-level, as there is no intermediate state between the trapped condensate (m_F=−1 Zeeman state) and the untrapped beam (m_F=0 Zeeman state). Fluctuations in the output will then be due to the back coupling and depletion of atoms to the antitrapped m_F=1 Zeeman state as observed previously [14]. However, in the limit that the output coupling becomes weak, these effects will be negligible and the system should produce a classically quiet atom laser beam. In Figs. 5 and 6 we show the population dynamics for the weak and strong coupling regimes for the three- and two-state systems, respectively. The behavior of the three- and two-state population dynamics is qualitatively similar to the five-state system. For weak coupling strengths the condensate decays monotonically, with a greater fraction of condensed atoms transferred to the m_F=0 state than the five-state system under similar conditions. In the strong coupling limit the behavior of the system is similar to the five-state, although here there is a cleaner cyclic oscillation of atoms between the three Zeeman states. Once again a percentage of all states remain trapped.
FIG. 7. (Color online) Periodic spatial oscillations in the population dynamics of a five-state system. One can see the ejection of the untrapped dressed states as well as both oscillation periods corresponding to the $F=2$ and $F=1$ radial trapping frequencies. Note that the upper edge of the oscillations is at the minimum of the magnetic trapping potential.

The oscillatory population dynamics observed in Figs. 1, 5, and 6(b) for the strong coupling limit is also reflected, if not driven, by periodic spatial oscillations. In Fig. 7 we show an example of such oscillations for the five-state system. In this figure one sees the ejection of the untrapped dressed states early in the simulation, and then a clean periodic oscillation. Two oscillation periods are clear from this figure, corresponding to the $F=2$ and $F=1$ radial trapping frequencies. Interestingly, these oscillations have their upper maximum positions at the minimum of the magnetic trapping potential (the point of gravitational sag for the $F=2$ state), and hence oscillate up only one side of the total potential (trap plus gravity). Similar behavior occurs for the two- and three-state systems as well. We believe that further detailed theoretical and experimental studies of these unusual dressed-state spinor (multicomponent) condensates will reveal a rich and potentially useful phase space. Already, dressed systems have been used to demonstrate two-dimensional trapping potentials [25] and to investigate double-well interferometry with condensates [26].

D. Comparison between two- and three-state systems

In this section we compare the dynamics of the two- and three-state atom lasers. As mentioned in the Introduction, significant effort is required in order to produce a “closed” two-state atom laser as the relevant alkali-metal atom manifolds have at least three Zeeman states linked by allowed rf transitions. Given that back-coupling fluctuations and the bound state arise even in the two-level system, limiting the homogeneous output flux as in the “natural” systems, it is prudent to ask whether the two- and three-state atom lasers actually differ that much in the weak coupling limit.

In Fig. 8(a) we compare the time dependent density of the two- and three-state atom lasers at a point below the condensate. It is clear that, for the 1D model we are using, the two systems are essentially indistinguishable in the weak coupling regime. It is interesting to note that even for stronger coupling, the details of the classical fluctuations on the beam are also mirrored in the two systems. The difference in amplitude between the simulations is accounted for by the loss of atoms to the antitrapped Zeeman state in the three-level system. In the weak coupling limit these atoms are expelled from the system on a time scale much faster than the back-coupling time set by the Rabi frequency, and so have little effect on the dynamics of the system.

In Fig. 8(b) we show the density fluctuations as a function of time for the two- or three-state atom laser. It is clear that even in the weak coupling limit taken here the density fluctuations due to back coupling are a significant contribution to the noise in the atom laser beam. This has major implications for the use of atom lasers in precision measurement systems, where high flux and minimal classical noise are essential features required of the atom beam.

In Fig. 9 we compare the number of atoms in the condensate to the number of atoms in the $m_F=0$ atom laser state for 15 ms of output coupling. There is a clear peak in the number of atoms transferred into the $m_F=0$ state as a function of Rabi frequency for all systems considered in this paper, with the two-state system having the highest coupling efficiency and the five-state having the lowest. However, a comparison of Fig. 9 with Fig. 8 reveals that we will not be able to operate the atom laser near the peak output-coupling rate because of increasingly severe density fluctuations, which impede the usefulness of the beam for measurement applications. For example, the peak in the two-state system is around $\Omega=500$ Hz while the density fluctuations are already significant at a Rabi frequency of 130 Hz.

In summary, we find the peak homogeneous output-coupling rate achievable in an atom laser to be significantly lower than the maximum output-coupling rate. For the 1D model considered here we find that, in this homogeneous output-coupling regime, there is practically no difference in

FIG. 8. (a) Comparison of the time-dependent density of the two- and three-state atom lasers at a point below the condensate after 15 ms of output coupling and for different Rabi frequencies. (b) The time-dependent density of the two-state atom laser at very low intensities after 100 ms of output coupling.
our results between the three- and two-state atom lasers. One may ask whether the small number of antitrapped atoms
generated in the three-state system will affect the results of a
correlation function measurement on the atomic beam, such
as the one made recently in the Zurich group [16]. In this
experiment an atom laser beam is produced in the weak cou-
pling regime and subsequently falls through a high finesse
cavity, in which the researchers are able to detect the passage
of a single atom with high temporal resolution. A simple
calculation shows that the small number of antitrapped atoms
produced by a three-state atom laser system would miss the
cavity by a large margin, thus having no effect on the results
of the measurement statistics for the $m_F=0$ state. One can
imagine many experiments where a true two-level system is
important, but we conclude that the atom laser is not one of
them, at least in the weak coupling limit.

E. Discussion of results with respect to future experiments

We plan to carry out further experiments in the near fu-
ture. By creating a very large condensate we should be able
to have both resonance positions (of the $m_F=2$ and $m_F=1$
trapped states) inside the condensate. One can expect the
spatial dynamics imposed on the atom laser beam by the
mechanism of the $m_F=1$ sloshing would then disappear.
However, preliminary simulations carried out in this confi-
guration seem to show no difference and further investigation is
necessary.

A Raman output-coupling setup will also be implemented
as it overcomes many problems with rf output couplers [27].
In particular, in the case of a Raman atom laser, we could
selectively outcouple atoms in a given Zeeman state, taking
advantage of the spatial dependence of the resonances.

IV. CONCLUSION

In this paper, we have characterized how rf output cou-
lping affects the spatial structure and temporal dynamics in a
BEC. We also presented the 1D mean-field model used for
this purpose and showed its trustworthiness by comparing it
to previous experimental measurements done in our lab. The
peak homogeneous output flux, classical fluctuations in the
atom laser beam, and the onset of a bound state were inves-
tigated in the cases of multistate and two-state atom laser
systems. The results showed the five-state system was clearly
inappropriate to create a clean and homogeneous atom laser
beam. It also appeared that the two- and three-state systems
were almost indistinguishable in the weak coupling regime
where the atomic beam is homogeneous, with no density
fluctuations. As a conclusion we think the “natural” three-
state system should be preferred as it does not require any
additional experimental devices.

We are currently studying a momentum kicked Raman
coupling based on the same model and it will be interesting
and important for future measurement devices to compare
the results to those described in the present paper.

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