Limits to the flux of a continuous atom laser

N. P. Robins,* A. K. Morrison, J. J. Hope, and J. D. Close
Australian Centre for Quantum Atom Optics, The Australian National University, Canberra, 0200, Australia
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We present experimental results demonstrating that the flux of a continuous atom laser beam cannot be arbitrarily increased by increasing the strength of output coupling. We find that the continuous atom laser has large density fluctuations when the Rabi frequency becomes greater than the radial trapping frequency, due to the output coupling populating all accessible Zeeman states. In addition, we find that a state-changing output coupler for a continuous atom laser switches off as the output-coupling strength, parametrized in general by the Rabi frequency, is increased above a critical value. The numerical solution of a one-dimensional five-state Gross-Pitaevskii model for the atom laser is in good qualitative agreement with the experiment.

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In precision measurement applications, atom lasers have the potential to outperform optical lasers and nonoptical techniques by many orders of magnitude, provided we can increase their flux, and achieve shot-noise limited operation techniques by many orders of magnitude, provided we can

 experiments with a 3-ms continuous atom laser, produced in the $F=2$ manifold of $^{87}$Rb by state changing output coupling to a magnetic field insensitive state, $m_F=0$. At low output-coupling strength, the atom laser beam flux increases gradually and homogeneously until $\Omega\approx1$ kHz, where $\Omega=g_F\mukB/2\hbar$ is the bare angular Rabi frequency. At around this value we observe that the antitrapped $m_F=-2,-1$ states begin to play a part in the atom laser dynamics. This leads to increasingly severe fluctuations in the density of the atom laser beam, and a loss of atoms to the antitrapped states [Figs. 1(a) and 1(b)]. A further increase in the coupling strength incrementally shuts down the output starting around $\Omega\approx4$ kHz. In the limit of large output-coupling strength (to be defined later), we find approximately 70% of the atoms remain localized in the condensate, while the other 30% are emitted shortly after the beginning of the output coupling period.

The rest of this paper is organized into three parts. First, we describe the details and relevant time scales of our experiment. In the second part we examine, experimentally, the regime of weak-to-intermediate output coupling. In the third

FIG. 1. Optical depth plot (35 independent experiments) showing the spatial structure of a 3-ms atom laser as a function of output-coupling strength. The system was left to evolve for a further 4 ms before the trap was switched off and 2 ms later the images were acquired. The field of view for each individual image is 0.7 × 0.3 mm. The final image is taken with $\Omega=16$ kHz. Antitrapped Zeeman states are clearly visible in (a), (b), and (c), which show an extended view (2.7×0.3 mm) of the data in the main part figure (represented by the dashed box).

*Electronic address: nick.robins@anu.edu.au

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FIG. 2. Outcoupled fraction in the Zeeman states of the $F=2$ single pulse (50 $\mu$s) atom laser as a function of angular Rabi frequency. Solid lines: theoretical calculation with no free parameters from a 3D GP model of the five-state $F=2$ atom laser.

part of the paper we demonstrate the “shut down” or bound state of the atom laser.

In our experiment, we produce an $F=2$, $m_F=\pm 2$ $^{87}\text{Rb}$ condensate, consisting of approximately $10^5$ atoms, via evaporation in a water-cooled QUIC magnetic trap [11] with a radial trapping frequency of $\nu_z=260$ Hz and an axial trapping frequency $\nu_z=20$ Hz. We operate our trap at a bias field of only $B_0=0.25$ G. The trap runs at 12 A (12 V) generated from a low noise power supply. The low power dissipation suppresses the heating-related drift of the magnetic trap bias field, allowing precise addressing of the condensate with resonant rf radiation (we measure a drift of significantly less than 0.7 mG over 8 h). Unless otherwise stated, in all experiments described in this paper the rf cut is placed 2 kHz from the upper edge of the condensate. After 50 s of evaporative cooling, the BEC is left to equilibrate for 100 ms. A rf signal generator, set in gated burst mode, produces a weak 3–200-ms pulse that is radiated perpendicular to the magnetic bias field of the trap through a 22-mm radius single wire loop, approximately 18 mm from the BEC. The peak rf magnetic field magnitude, $B$, and hence the bare Rabi frequency, $\Omega$, is calibrated by fitting the frequency axis of a $50 \mu$s pulsed experiment with a numerical simulation of pulsed output coupling utilizing the three-dimensional (3D) Gross-Pitaevskii (GP) theory of the atom laser described in our previous paper [7]. The results of such a calibration are shown in Fig. 2. We find that the output-coupling magnetic field seen by the atoms is reduced by around a factor of 5 compared to a simple calculation of the field radiated from a single loop or a free space measurement of the field from the coil. This reduction is due to the close proximity of the quadrupole trapping coil on which the rf coil is mounted. Using this method we estimate the uncertainty in the Rabi frequency to be around 3–5%. The quadratic contribution of the Zeeman effect creates resonance shifts of approximately 0.02% at 1 G, and hence our system is modeled accurately by the five-state GP equation.

To put this work into context, it is critical to understand the time and length scales relevant to the operation of a state changing output coupler. In the Thomas-Fermi (TF) limit the addressable frequency width of a condensate in the $F=2$, $m_F=\pm 2$ state is given by $W=(g/\hbar \omega_\perp)^2 M \mu$, where $\mu$ is the chemical potential, $M$ is the atomic mass, $g$ is gravity, and $\omega_\perp$ is the radial trapping frequency [5]. (For our condensates $W=2\pi \times 5$ kHz.) For the continuous atom laser the resonance width of the output coupling is determined by the Rabi frequency, $\Omega$, due to power broadening. A number of definitions exist in the literature for what constitutes weak output coupling, all based on a comparison of the coupling strength, $\Omega$ with various parameters. All require that the resonance width of the output coupling is significantly less than the frequency width of a condensate. The most stringent of these definitions requires $\Omega \ll \omega_\perp \sim 1.6$ kHz [12,13]. A more relaxed condition states that $\Omega \ll 1.66 \sqrt{g/R} \sim 3.2$ kHz, where $R$ is the spatial Thomas-Fermi width [14]. The least stringent condition is $\Omega \ll 1/\tau \sim 12$ kHz, where $\tau$ is the time it takes atoms to leave the output-coupling resonance [15]. Strong output coupling can be considered to occur when the weak output-coupling conditions are not met.

In Fig. 3 we display experimental data taken under a similar set of parameters to Bloch et al. [5]. A 10-ms atom laser beam is produced and allowed to separate from the condensate prior to imaging. The decline of the trapped condensate atom number clearly follows a simple monotonic decay as found by the previous work [5]. However, from this data it can be seen that the decline of the population in the trapped states does not lead to a monotonic increase in the population of the $m_F=0$ state that forms the atom laser beam. Our conclusion is that the missing atoms have been expelled into the antitrapped states. We find the same behavior for all output-coupling times that produce a fully observable atom laser beam (up to around 25 ms for our imaging field of view). For shorter output-coupling times, such as those presented in Fig. 1, we are able to observe the antitrapped states, which appear above an angular Rabi frequency of $\Omega \approx 800$ Hz. For all our data, we find that the peak output flux occurs at $\Omega \approx 1$ kHz, and that this value corresponds to the most homogeneous atom laser beam. As $\Omega$ is increased above 1 kHz the atom laser beam develops steadily worsening density fluctuations, the most prominent feature of which is a peak in the leading edge of the beam. One way around this type of flux limiting dynamics is to reduce the atom laser system to only two levels: trapped and untrapped. Such a system has been demonstrated by the Aspects group, utilizing the nonlinear Zee-
man shift in the F = 1 manifold to create an effective two-level system. A Raman transition directly from trapped to untrapped states with a two-photon momentum transfer should also increase the homogeneous flux limit as it removes atoms more quickly from the output-coupling region. However, in what follows we show that for all continuous state changing output coupling, including two-state systems, output flux is limited by the underlying physics of the process. 

In Fig. 4 we display a measurement of the number of atoms remaining in the trapped states as the Rabi frequency is scanned from the weak-to-strong output-coupling regime for a 100-ms continuous atom laser (open circles in the figure). Note that for this output coupling time the peak flux in the atom laser beam is below our detection sensitivity. The condensate is progressively depleted at increasing, but weak, Rabi frequencies (Fig. 4, inset) until we observe no atoms remaining in the magnetic trap. At Ω = 4 kHz atoms start to reappear, and by Ω = 9 kHz the number of atoms remaining in the trap stabilizes to 70% of the initial condensate number. As described earlier, the other 30% are expelled at the initial switch on of the output coupling. Our method of collecting data ensures that there are no systematic shifts in the trap bias field. Every 10 runs of the experiment we switch to a 10-ms atom laser to ensure constant detuning, as well as comparing detailed data to a “coarse” set taken at the beginning of each run. We emphasize here that the Rabi frequencies over which this effect occurs correspond to the crossover from the weak-to-strong coupling regime. In order to measure atoms remaining in the BEC after 100 ms of output coupling, the combination of the magnetic trap and rf field in the crossover regime must be creating a trap for the atoms. Equivalently, the atom laser ceases to operate, or shuts down in the intermediate-to-strong coupling regime. We have verified that the shut-down edge is independent of output-coupling duration, as shown in Fig. 4 with a 200-ms atom laser (black dots). For shorter coupling times the condensate is not fully depleted before this edge is reached (see, for example, Fig. 1).

A number of theoretical works have suggested that the atom laser may have a bound eigenstate [8–10], based purely on the existence of coupling between a single trap mode and a continuum of untrapped states. Furthermore, in the context of producing a two-dimensional BEC, it has been shown recently that trapping of all m_F states is a natural consequence of combining rf coupling with a dc magnetic trap [16,17]. This trapping can be understood by considering the “dressed state” basis in which the rf coupling and dc potentials seen by the atoms are diagonalized. In this basis the dressed eigenstates are linear combinations of the bare Zeeman states, trapped in effective potentials created by the avoided crossings. Assuming the strong coupling limit and a sudden nonadiabatic projection onto the dressed states, diagonalization yields a prediction of up to 62.5% of the initial condensate atoms remaining trapped for the F = 2 atom laser (four of the five dressed states allow some trapping). Our experimental observation of 70(±7)% is consistent with these values, if slightly higher, suggesting some degree of adiabatic transfer. In order to gain further qualitative insight we numerically solve the F = 2 Gross-Pitaevskii model [7] of the atom laser in one dimension given by

\[
i\dot{\phi}_2 = (L + z^2 + Gz - 2\Delta)\phi_2 + 2\Omega\phi_1,
\]

\[
i\dot{\phi}_1 = \left(L + \frac{1}{2}z^2 + Gz - \Delta\right)\phi_1 + 2\Omega\phi_2 + \sqrt{6}\Omega\phi_0,
\]

\[
i\dot{\phi}_0 = (L + Gz)\phi_0 + \sqrt{6}\Omega\phi_1 + \sqrt{6}\Omega\phi_{-1},
\]

\[
i\dot{\phi}_{-1} = \left(L - \frac{1}{2}z^2 + Gz + \Delta\right)\phi_{-1} + 2\Omega\phi_{-2} + \sqrt{6}\Omega\phi_0,
\]

\[
i\dot{\phi}_{-2} = (L - z^2 + Gz + 2\Delta)\phi_{-2} + 2\Omega\phi_{-1},
\]

where φ_i is the GP function for the ith Zeeman state and \(L = -(1/2)\left(\partial^2/\partial z^2\right) + U(\Sigma_{m_F = -2}^2|\phi_i|^2)\). Here Δ and Ω are, respectively, the detuning of the rf field from the Br resonance, and the Rabi frequency, measured in units of the radial trapping frequency \(\omega_z\) (for the m_F = 1 state), U is the interaction coefficient and G = (mg/\hbar^2)(h/m_0)^{1/2} gravity. The wave functions, time, spatial coordinates, and interaction strengths are measured in units of \((\hbar/m_0)^{-1/2}, \omega_z^{-1}, (\hbar/m_0)^{1/2}\), and \((\hbar/\omega_z)^{-1}(\hbar/m_0)^{-1/2}\), respectively. The nonlinear interaction strength is derived by requiring that the one-dimensional (1D) TF chemical potential be equivalent to the 3D case. There are no free parameters in this model; we use \(U = 6.6 \times 10^{-4}, G = 9.24, \Omega = 0 - 13.85, \Delta = 8.2\).

Figure 5(a) shows the time evolution of the population numbers in the trapped, untrapped and antitrapped Zeeman states for \(\Omega = 800\) Hz. The trapped \(m_F = 2,1\) populations tend towards zero, essentially monotonically. In Fig. 5(b), corresponding to \(\Omega = 16\) kHz, we find an entirely different behavior. After an initial high-frequency exchange of the number of atoms in the different Zeeman states, slightly more than 50% of the atoms are ejected from the system. The remaining atoms, which populate all zeeman states, undergo a lossless and slow exchange of population. We identify the trapped atoms as being in a superposition of the dressed state
FIG. 5. Theoretical results obtained by solution of the 1D-GP equation. The atomic population in the Zeeman states as a function of time for a 15-ms atom laser for (a) $\Omega=800 \text{ Hz}$ and (b) $\Omega=16 \text{ kHz}$. In (c) we show the behavior of a pure two-state atom laser system for $\Omega=16 \text{ kHz}$. (d) The total atomic population remaining in the magnetic trap after 100 ms of output coupling plotted as a function of angular Rabi frequency.

The ejected atoms simply correspond to the untrapped or antitrapped dressed states. Figure 5(c) shows the same trapping effect, but this time in a simple two-level atom laser system, indicating that the bound mode is inherent to state changing output coupling regardless of the number of states involved. Finally, Fig. 5(d) shows a prediction of the number of atoms remaining in the trap after 100 ms of output coupling as a function of Rabi frequency. The behavior qualitatively matches that of the experiment, indicating that we are indeed observing the bound state of the atom laser in our experiments. The predicted number of atoms remaining in the trap lies within the range of the strong coupling estimate, but still differs substantially from our experimental results. We expect that a full 3D model would capture the dynamics of our experiment, but it is beyond the scope of the present paper.

In conclusion, we have shown that the peak homogeneous output flux of an atom laser beam derived from a finite BEC is limited by the requirement to operate well within the weak coupling regime, in order to avoid pumping to antitrapped states and the related density fluctuations. An obvious short term solution to boost the flux would be to chirp the rf detuning across the condensate; however, such a scheme would not be applicable to a pumped continuous atom laser. Additionally, we find that one cannot arbitrarily increase the output flux of a state changing atom laser due to the presence of bound modes. We are currently investigating momentum kicked continuous Raman coupling and condensate pumping as a means of boosting atom laser flux.

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