Ion Cyclotron Production by a Four-Wave Interaction with a Helicon Pump

O. Sutherland, M. Giles, and R. Boswell

Plasma Research Laboratory, RSPhysSE, Australian National University, 0200, Canberra, Australia

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Ion heating mechanisms play an important role in a number of fusion and laboratory experiments and are also being exploited in novel space propulsion systems for interplanetary missions [1]. Typically, ions are heated by some form of wave-particle interaction either through direct coupling with an antenna or indirectly through the decay or resonant interaction of a higher frequency pump wave with an ion wave. The most common resonant heating process reported in fusion devices is direct coupling via ion cyclotron resonant heating (ICRH) [2–4], although direct coupling through lower hybrid [5], Alfvén [6], upper hybrid [4], and ion Bernstein waves [7] have also been investigated. ICRH is also the favored mechanism for ion heating in NASA’s VX-10 plasma rocket concept where it is proposed that the increased perpendicular ion energy from the resonant interaction is converted into thrust via the adiabatic expansion of a magnetic nozzle [8].

Space plasma experiments are more focused on naturally occurring plasma instabilities as channels for energy transfer to ions. Kindel and Kennel showed that electrostatic ion cyclotron waves (EICW) are preferentially driven by magnetic field aligned currents under ionospheric plasma conditions and therefore could account for ion cyclotron spectral features as well as increased ion temperatures in space plasma measurements [9]. More recently, Spangler et al. [10] and Scime et al. [11], extending the seminal work of Ganguli et al. [12] and the subsequent development of Gavrishchaka et al. [13] on parallel flow shear driven ion acoustic and ion cyclotron waves, showed that ion temperature anisotropy \( T_{ii} \ll T_{ik} \) could further reduce the threshold current for these instabilities. Experimentally, Teodorescu et al. [14] and Keopke et al. [15] have reported evidence of electrostatic ion cyclotron waves in the presence of parallel velocity shear and temperature anisotropies. In this Letter we report on the experimental observation of EICW via a four-wave resonant interaction with a helicon wave. The experimental results are explained in terms of a filamentation type instability where the helicon pump couples into the electrostatic ion cyclotron noise field, creating fluctuations in density that refract the pump. This produces two daughter helicons which propagate at a small angle to the ambient magnetic field and beat at about 0.7 times the ion gyrofrequency in a positive feedback process.

Experimental measurements were undertaken in the large volume helicon reactor waves on magnetized beams and turbulence (WOMBAT). The device is described extensively elsewhere [16], but briefly consists of a 50 cm long, 18 cm inner diameter Pyrex source tube terminated at one end by grounded aluminum plate and attached co-axially at the other to a 200 cm long, 90 cm inner diameter grounded stainless steel chamber. It is equipped with a number of solenoids but only those three internal to the diffusion chamber were used in the experiments presented here, resulting in a field uniformity of 0.5% along the axis of the diffusion chamber and a convergent field in the source region. Argon feed gas was introduced with sufficient flow rate to maintain 1.2 mTorr of working pressure, and 1.9 kW of cw radio frequency (rf) power was provided at 7.2 MHz, via a matching network, to a double half-turn antenna axially aligned around the source, 7 cm from the end plate. Under these operating conditions and a magnetic field of 130 G, a plasma density in excess of \( 10^{13} \, \text{cm}^{-3} \) was obtained in the source (as indicated by the cutoff of a 35 GHz interferometer) which, in conjunction with the magnetic field geometry, produced a column along the whole length of the diffusion chamber, roughly 8 cm in diameter, radiating strongly on the 488 nm Ar II line (and relatively little on the 911 nm Ar I) resulting in a powerful blue glow.

All of the experimental measurements presented here were taken using a shielded Langmuir probe with two 5 mm long, 0.25 mm diameter tungsten tips separated by 3 mm. The probe assembly could be translated radially and the probe tips were aligned to be in the plane containing the reactor axis of symmetry. The spectral measurements were executed by connecting the upstream probe tip to the 50 \( \Omega \) input of a heterodyne spectrum analyzer via a dc block (a small capacitor) and windowing the output signal 15 times using a 12-bit ADC. These windows were then averaged to increase the signal-to-noise ratio of the measurements. The probe was introduced at approximately 50 cm from the junction between the source and the diffusion chamber, representing a distance of about 93 cm from the antenna.
A typical set of spectra, measured here at a distance of 30 mm from the center of the column for a magnetic field of 158 G, is shown in Figs. 1 and 2. The helicon wave (7.2 MHz) has clear upper and lower sidebands at 5 kHz from the center frequency, and a low frequency wave exists at 5 kHz. Sidebands at two and three times the ion wave frequency were also occasionally observed. The low frequency wave was often less distinguishable from the background noise than the helicon sidebands, its amplitude depending heavily on the correct tuning (not necessarily the lowest reflected power), and was rarely accompanied by waves that could be interpreted as harmonics. These waves were highly localized in the center of the column, as can be seen from Fig. 3 with highest amplitudes for radii less than 10 mm. It was also observed, however, that the broadband noise was much higher in this region, reducing the signal-to-noise ratio considerably.

Both the upper and lower sidebands follow the same evolution as a function of radius decreasing monotonically upon moving away from the center. The low frequency wave as well as all of the sidebands were observed to change frequency as a function of magnetic field. Figure 4 shows that the absolute value of the difference between the first harmonic upper and lower sideband frequencies and the helicon frequency for magnetic fields between 130 and 158 G is roughly linear and sits on a line 0.7 times the theoretical ion gyrofrequency. No measurements below 130 G were taken because both the low frequency wave as well as the helicon sidebands could not be distinguished from the background noise for these magnetic fields. As the frequency of the ion wave was close to the theoretical ion gyrofrequency and was a linear function of magnetic field, it is proposed that this wave was an ion cyclotron mode with a lower bound threshold magnetic field of 130 G. An upper bound on the magnetic field could not be determined as the field in the diffusion chamber was limited to 158 G by the power supply.

As indicated above, the experimental results can be explained in terms of a four-wave parametric instability of the filamentation type. The following physical argument shows how it is possible for a helicon to couple into the ion wave noise field, in the region of gyromagnetic resonance, in a manner which can amplify a density fluctuation in the frequency range slightly below \( \Omega_i \).

When the Alfvén speed \( V_A \) is much smaller than the speed of light \( c \), the low frequency branch of the electrostatic ion wave field \( \Omega_e \) can only differ appreciably from \( \Omega_i \) if the angle of propagation \( \theta \) to the external magnetic field \( \mathbf{B}_0 \) satisfies the condition \( \cos \theta \leq \sqrt{\mu} \), where \( \mu \) denotes the ratio of the electron and ion masses, \( \mu = m_e/m_i \). The dispersion relation is then
The essential elements of this physical picture can be described in terms of Maxwell’s equations together with the cold fluid equations for the ions and electrons, from which coupled equations for \( \mathbf{B} \) and \( \delta n \) can be derived as follows. For the helicon, \( \mathbf{B} \) satisfies the well known linear wave equation, supplemented here by the nonlinear term which represents the perturbation to the refractive index, so that

\[
(n_0 + \delta n) \frac{\partial \mathbf{B}}{\partial t} = -\frac{c^2 \Omega_e}{\omega_p^2} n_0 (\mathbf{b} \cdot \nabla) \text{curl} \mathbf{B},
\]

(2)

where \( \omega_p \) and \( \Omega_e \) are the electron plasma and gyrofrequencies, while \( \mathbf{b} \) denotes a unit vector parallel to \( \mathbf{B}_0 \). According to the physical model, the only nonlinear term that needs to be taken into account in deriving the equation for \( \delta n \) is the mean Lorentz force \( \mathbf{F} \) acting on the electrons. In the case of the ions, the nonlinear coupling is represented indirectly through the potential \( \phi \) of the space charge field.

Thus, for the ions, allowing for their motion across \( \mathbf{B}_0 \) and linearizing their equations of continuity and momentum, we get

\[
\left( \frac{\partial^2}{\partial t^2} + \Omega_i^2 \right) (\mathbf{b} \cdot \nabla)^2 \frac{\delta n}{n_0} = -\frac{e}{m_i} \nabla^2 (\mathbf{b} \cdot \nabla)^2 \phi.
\]

For the electrons, allowing for their flow along \( \mathbf{B}_0 \) and using their linearized equations of continuity and momentum, supplemented by the mean Lorentz force, we get

\[
\frac{\partial^2 \delta n}{\partial t^2} n_0 = -\frac{e}{m_e} (\mathbf{b} \cdot \nabla)^2 \phi - \frac{1}{m_i n_0} \mathbf{b} \cdot \nabla (\mathbf{b} \cdot \mathbf{F}).
\]

Eliminating \( \phi \) from these two equations and substituting for \( \mathbf{F} \) leads to

\[
\left\{ \left( \frac{\partial^2}{\partial t^2} + \Omega_i^2 \right) (\mathbf{b} \cdot \nabla)^2 + \mu \frac{\partial^2}{\partial t^2} \nabla^2 \right\} \frac{\delta n}{n_0} = -\nabla^2 (\mathbf{b} \cdot \nabla) \left( \frac{1}{B_0^2} \mathbf{b} \cdot \text{curl} \mathbf{B} \times \mathbf{B} \right)
\]

(3)

A dispersion relation for the parametric instability can be derived from (2) and (3) by a standard plane wave analysis in which the solution is approximated as a four-wave interaction involving the coupling of a helicon pump wave to an ion wave through upper and lower sideband waves. The frequencies and wave numbers of the waves involved are as follows (referred to rectangular coordinates with \( \Omega_i \) parallel to \( \mathbf{B}_0 \): (a) the pump wave, \( \omega_0 \), \((0, 0, k)\); (b) the upper and lower sideband waves, \( \omega_{\pm} \), \((l, 0, k \pm m)\); and (c) the ion wave, \( \Omega_i \), \((l, 0, m)\). The familiar helicon dispersion relation follows from (2), giving \( \omega_0 = k^2 c^2 \Omega_e / \omega_p^2 \) and \( \omega_{\pm} = \omega_0 (k \pm m)^2 / \ell^2 / k \). Since \( m \ll l \ll k \), we have \( \omega_{\pm} \approx \omega_0 = \omega_0 (1 + \ell^2 / k^2) \), so that the difference frequency \( \Omega_i \approx \omega_1 - \omega_0 \approx \omega_0 q^2 / 2 k^2 \). In addition, (3) yields the dispersion relation (1) with \( \tan \theta = l / m \).

The result of the nonlinear stability analysis based on (2) and (3) is the parametric dispersion relation

\[
(\Omega^2 - \Omega_i^2)(\Omega^2 - \Omega_p^2) = -\Omega_i^2 (P \Omega_1 + Q \Omega),
\]

(4)

where
\[ P = C \frac{km^2l^2}{k^3(m^2 + \mu l^2)} \]
\[ Q = C \frac{m^4}{2k^3(m^2 + \mu l^2)} \]

in which
\[ C = \frac{\omega_0}{\Omega_r} \left( \frac{\omega_p E_0}{\Omega_i B_0} \right)^2. \]

where \( E_0 \) is the amplitude of the electric field of the pump wave.

Instability is associated with the terms on the right side of (4) which exert their greatest effect on the ion wavelength at which their coefficients are largest. Clearly, by (5), \( Q \) has a maximum when \( m/l = \sqrt{\mu} \), which is in accord with the condition for the ion wave to propagate. In these circumstances, by (1), \( \Omega_i \) assumes the value \( \Omega_{i0} = \Omega_i / \sqrt{2} \). It will be noticed that \( P \) has only reached half its maximum value. But this is to be expected because its full maximum as a function of \( m \) is only reached when \( m^2 \gg \mu l^2 \), which cannot be realized without violating the conditions for the existence of the ion wave.

It can now be demonstrated that \( \Omega_{i0} \) is, in fact, the frequency of the most unstable wave. To do this, it is sufficiently accurate to substitute the real part of \( \Omega \) in the small correction terms on the right side of (4), which we then write as \(-\Omega_0^r\). Solving for \( \Omega^2 \) now yields
\[ \Omega^2 = \frac{\Omega_{i0}^2 + \Omega_r^2}{2} \pm i \left( \Omega_{i0}^4 - \left( \frac{\Omega_{i0}^2 - \Omega_r^2}{2} \right)^2 \right)^{1/2}. \]

This result shows that the growth rate is largest when
\[ \Omega_{i0} = \Omega_r. \]

We thus confirm that the frequency of the most unstable wave is given approximately by
\[ \Omega_{i0} = \frac{\Omega_r}{\sqrt{2}} \approx 0.7 \Omega_r. \]

Equation (6) is a resonance condition which states that the linear helicon waves have different frequencies which match the ion wave frequency. However, the nonlinear waves have frequency shifts which displace the sideband frequencies to \( \omega_0 \pm \Omega_i \). Note further that (6) yields the transverse wave number of the ion wave as \( l = k \sqrt{2 \Omega_{i0}/\omega_0} \), which confirms that the required conditions for (1) to hold, \( m \ll l \ll k \) and \( l/c/\Omega_i = 2^{1/4}c/V_A \gg 1 \), are satisfied by this approximate solution.

We have not of course taken account of the cylindrical geometry of the actual device in the foregoing, in an effort to simplify the nonlinear analysis. However, it is argued that it would not alter our main result (7), because the key features of the theory leading to (7) would retain their present form. In particular, (1) and (4) remain valid in cylindrical geometry, as does the dependence of \( P \) and \( Q \) on \( m \) and \( l \), as given in (5), except that \( l \) becomes a radial wave number. Similarly, the resonance condition (6) remains as the condition for maximum growth. The expression for the pump power coefficient \( C \) would change significantly in cylindrical geometry, but we only use the fact that \( C > 0 \) in deriving (7), so these changes are not relevant. Likewise, the complexity of the solution for the wave numbers would also increase significantly in cylindrical geometry but again, their actual values do not affect (7).

The helicon wavelength (in meters) is approximately \( \lambda = 2.2 \times 10^{10} \sqrt{B/\nu f} \), where \( B \) is in Gauss, \( n \) in \( m^{-3} \), and \( f \) in Hz. For 158 G and \( 10^{19} \ m^{-3} \) we have 0.0033 m and a wave number of \( 1.9 \times 10^2 \) rad \( \cdot m^{-1} \). The transverse wave number for the helicon sidebands was \( l = 10 \) rad \( \cdot m^{-1} \) so that the angle of propagation was about 3° (satisfying the condition \( m \ll l \ll k \)). The associated wavelength for the ion wave was 63 cm, which corresponds well with the inner diameter of the diffusion chamber. The column was close to full ionization with a density of \( 10^{13} \ cm^{-3} \), which for a magnetic field of 158 G yielded an Alfvén wavelength of roughly 2 m. Given that both the longitudinal and transverse wavelengths correspond to the diffusion chamber dimensions, the EICW could be seen as a particular resonance condition of our high density, low magnetic field experimental apparatus.