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Grain-based characterisation and acoustic wave propagation in a sand packing subject to triaxial compression

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Abstract. This paper presents a study of 3D deformation process in a dry packing of Ottawa sand. X-ray microtomography is used to acquire scans of a triaxial test of the sample at five axial stress levels. Using 3D image analysis we are able to resolve particle scale features. Particle tracking combined with finite element simulations reveal that the rotational transformation of particles is one of the primary mechanisms of elastic energy dissipation at the grain scale. By analysing grain contact orientation, we show that stress induced anisotropy is spatially correlated to the compressional elastic wave.

Keywords: x-ray computed tomography, sand packing, triaxial compaction, finite element simulations.
PACS: PACS: 45.70.Cc; 45.70.-n; 45.70.Qj

INTRODUCTION

Characterisation of compaction process in granular systems is a multi-scale problem. At the micro level, the geometry of particles and the interaction between them which is dictated by the the contact properties are the major players. At the macro scale the system’s boundary conditions (whether stress or strain controlled) influences many of it’s effective mechanical properties.[1, 2, 3]. A large part of the problem arises from the strongly non-linear contact law between continually rearranging rigid bodies coupled with the dynamical nature of the contact network. In this context, the spatial distribution and temporal evolution of the force network in the bulk is a chief quantity that expands from micro (grain-grain contacts) to macro scale (grain assembly). Many studies have attempted to use complex models that account for elastoplastic deformation and non-affine displacement at the grain scale to bridge between micro and macro scale [4, 5, 6, 7]. Recently, x-ray computed tomography (XCT) has been utilised to quantify sheared soils and compaction of granular systems [8, 9, 10, 11].

In this study we utilise XCT to observe the deformation process in a packing of Ottawa sand, confined in a cylindrical triaxial compression cell. Using 3D image processing tools to track individual grains and an implementation of Discrete Digital Image Correlation (DDIC) we map volumetric strain evolution of the compacting structure. Additionally we investigate the correlation between grain contact orientation and its effect on compressional elastic wave during the compaction process by combining the finite element method (FEM) with the tomographic data.

EXPERIMENT AND IMAGING

Triaxial compression tests were performed on packings of dry unconsolidated sandstone of pure quartz grains (Ottawa sand F30) packed in a cylindrical geometry of 7.5mm diameter and 17.6mm height. The sample comprises of roughly 4,000 sand particles and the grains have been sieved to achieve diameters ranging from 500 μm to 1mm. A novel sleeving technique has been used to create the packing of grains. The initial packing was prepared by applying a pre-loading protocol to reach a reproducible initial condition (low packing density). We choose a stress controlled boundary condition for loading procedure of this experiment: [12].

The pressure apparatus used in this study has been built to enable in-situ acquisition of tomographic images of rocks samples under triaxial mechanical loading. The stress-controlled test of Ottawa sand sample was performed under the confining pressure of 290 psi. A series of axial pressure steps at 900, 1620, 2340 and 3080 psi (marked as S1, S2, S3 and S4 in Figure 1(a)) were applied to one end of the packing at the confining pressure. XCT scans were taken at these key axial pressures throughout the test as indicated in Figure 1(a) with S0, S1, S2, S3 and S4. The transition between the steps of axial pressure is achieved by a ramp protocol of 0.1 psi/sec that lasts for 10 minutes before reaching the next stage of compaction (see Figure 1(a)). Once desired axial pressure is reached, we let the sample (sand packing) to relax for 6 hours before the tomogram acquired (12 hours). Axial pressure is kept constant during the image acquisition as seen in Figure 1(a).

Figure 1(b) shows the XCT setup and the pressure cell mounted between the x-ray source and CCD sAntilator.
The tomograms are of high quality (high signal to noise) and we are able to achieve a spatial resolution (voxel size) of 6.8 μm. Figure 1(c) shows a vertical slice through the tomographic image of the sample at stage 2 (axial pressure 1620 psi). It is worth mentioning that by visual inspection of tomograms, we observe grain breakage of about 1% only at the last stage of compaction.

PARTICLE TRACKING AND DELAUNAY-BASED DIGITAL IMAGE CORRELATION

The high resolution tomographic images combined with relatively large particle sizes (an average of about 100 voxels in diameter) allows for particle tracking in subsequent tomograms. We can then quantify each particle’s spatial changes by calculating translational ($\Delta x, \Delta y, \Delta z$) and rotational ($\Delta \theta, \Delta \phi$) transformation of each particle [13] at different stages of compaction:

$$
\begin{align*}
\Delta x_n &= x_n - x_0, \quad \Delta y_n &= y_n - y_0, \quad \Delta z_n = z_n - z_0 \\
\Delta \theta_n &= \theta_n - \theta_0, \quad \Delta \phi_n = \phi_n - \phi_0
\end{align*}
$$

where $x$, $y$ and $z$ are the cartesian coordinates of the particle’s centre and $\theta$ and $\phi$ define the particle’s orientation with respect to the $z$ axis (loading direction) and in the $x-y$ plane respectively. In the above notation, subscript $n$ denotes the $n$th stage of compaction and subscript 0 denotes the initial uncompressed stage. The set of particle transformations is sufficient for characterisation of particle-particle interaction.

With the knowledge of every particle’s position in each dataset of the compression stage, we can then follow changes in local volumetric elements. This is known as Digital Image Correlation (DIC) which is a technique that maps an image onto another. This method has gained popularity in characterisation of kinematics of evolving structures for its ability to measure bulk deformation and strain fields as well as volumetric strain changes [14, 15, 14]. We implement a discrete DIC (D-DIC) based on tracking of each grain throughout the compaction process at imaging intervals S0-S1, S1-S2, S2-S3 and S3-S4. Using particle centres (their centre of mass) as input data, we generate the Delaunay triangulation [16] of the 3D point set. The result is that the 3D volume occupied by particles is decomposed into tetrahedral elements whose vertices are the particle’s centres. By following the volume changes of each tetrahedron throughout the compaction process, we are able to map local porosity changes in the bulk throughout the compaction process. Figure 2(f) shows a 3D visualisation of local porosity increase of more than 10% in imaging intervals of S0-S4. This local volume dilation is often the result of geometrical and mechanical frustration that...
FIGURE 3. Young’s modulus calculated using FEM (red curve) and measure experimentally (black curve).

leads to the build up of stress locally. In Figure 2(g) we show a 2D slice of the 3D deformation field calculated based on the D-DIC. It is clear from the figure that there is no shear band (we check the 3D field as well) instead the deformation is rather localised. Most of the localised deformations are as a result of translational and/or rotational transformation of the sand grains. However, we observe that some are related to the onset of grain breakage seen mainly at the last stage of compaction.

FINITE ELEMENT ELASTIC SIMULATIONS

We measure elastic properties of our sand packings at each stage of compaction both experimentally and also using the finite-element methods (FEM). The FEM simulation outputs the full tensorial stress response of the samples including the Young’s modulus along the loading direction ($Y_z$). Figure 3 shows the evolution of $Y_z$ throughout the compaction at different axial pressures as indicated by horizontal axis. Initial bulk and shear moduli of 38 and 44 GPa (Young modulus of 95 GPa) were assigned to the solid phase (quartz), and the simulations were performed on full datasets of each stage of compression. The Young’s modulus measured experimentally from the stress-strain curve (Figure 1(a)) can only extract $Y_z$ for increment stages of $S_1 - S_2$, $S_2 - S_3$ and $S_3 - S_4$ and the values are in good agreement with simulations (or the other way!) as seen in Figure 3.

Although the elastic moduli that is calculated as standard output from FEM analysis provide valuable insights into the overall elastic properties but it does not reflect the micro scale particle interaction that leads to such effective properties. Indeed, in order to bridge between the micro and macro scale a better understanding of the underlying mechanisms at grain scale is essential. By combining FEM and our ability to track particles through-out the experiment, we are able to follow the evolution of elastic energy (normalised by grain volume) in each grain. Changes in the elastic energy of particles is a consequence of particle-particle interaction via their contacts, local structural motifs and the boundary conditions of the system. There are a number of grain scale mechanisms that allow for the relaxation of granular systems through the release/dissipation of the elastic energy of the particles. No significant shear is observed in this study but rotation and translation are abundant. The proportion of grains experiencing loss of energy is 18% of the total number of grains. This group of grains that significantly reduce their elastic energy show a clear trend with rotations larger than 5° (see Figure 4).

CONTACT DISTRIBUTION AND ANISOTROPY

The compaction process can induce structural anisotropy which can affect a range of macroscopic physical properties. Propagation of the compressional wave velocity ($V_p$) is one of such properties that travels through the medium (via contacts) and is sensitive to structural anisotropy. Structural anisotropy can be considered at different length scales and exactly how it can alter $V_p$ is still an open debate.

An efficient way of visualising the particle orientation and grain contact orientation distribution is to use hemispherical plots which is a two-dimensional histogram in a curved orientation space. The hemispherical surface was divided into approximately 30 bins along each of about 30 equiangular azimuthal segments, giving a total of 900 separate bins in the hemisphere [17]. The latitudinal extents of the bins were calculated so that all subtended equal solid angles on the surface. Figure 5 (top row) is the visualisation of grain orientation for each stage of
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comparing (left to right). These plots can be viewed via a rotatable 3D perspective on computer but here we show the orthographic projections viewed from above, with the vertical direction at the centre of the circular plot. To visualise the grain contact orientation (Figure 5(middle row)), we map the normal of the contact planes in 3D. The FEM simulation allows one to generate amacroscopic stress-strain tensor of the sample. We then use Christoffel’s equations to relate bulk stress-strain tensor properties in the sample to the velocity field. This velocity field can also be visualised using hemispheric plots as seen in Figure 5(bottom row).

Comparing the orientation and velocity distributions of Figure 5, there is a clear qualitative correlation between grain contact distribution and $V_p$ whereas grain orientation distribution does not exhibit such correlation [17]. This suggests that surface contact area normal to wave direction is important in facilitating transmission of longitudinal waves through the sample.

CONCLUSION

In this paper we have provided an overview of work in progress to understand the deformation mechanism of a packing of Ottawa sand subject to a series of axial loading. We have attempted to characterise the evolving structure from both geometrical and mechanical perspective by using advanced 3D image processing, particle tracking, D-DIC and FEM simulations.

We implemented a novel discreet digital image correlation based on the Delaunay triangulation of particle centres. We showed that local volumetric dilatancy (porosity increase) is highly localised and increases with axial loading.

We demonstrated that FEM analysis can accurately predict the effective elastic properties of the structure. By combining FEM and particle tracking, we showed that grains with high degree of rotation ($\sim 5^\circ$) loose up to 20% of their elastic energy.

High resolution tomograms allow us to spatially map grain contact distribution. We showed that the anisotropy induced during the compaction process is captured by grain contact distribution which determines the propagation direction of compressional elastic wave.

A small portion of grains (about 1%) break/crush during compaction. In future works, we plan to take into account grain breakage and its effect on both micro and macro scale properties.

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