Electric dipole moments from lattice QCD

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Outline

➤ Motivation
➤ Lattice QCD
  ➤ Brief intro
  ➤ Including $\theta$ term
  ➤ ...
➤ Neutron EDM
➤ Quark EDM, cEDM
➤ Outlook
Introduction

➤ Electric dipole moment $\vec{d}$
  ➤ violates P and T symmetries
  ➤ CPT invariance $\rightarrow$ CP violation

➤ Neutron EDM:
  ➤ A measure of physical separation of positive and negative charges within neutron
  ➤ Not yet observed!
  ➤ Current bound

$$|d_n| < 3.0 \times 10^{-26} \text{ e . cm}$$
Introduction

➤ CP violation in standard model:
   ➤ CKM matrix:
      
      neutron: $|d_n| \sim 10^{-31} \ e \cdot \text{cm}$
      electron: $|d_e| \sim 10^{-38} \ e \cdot \text{cm}$

➤ Strong CP ($\Theta$ term):

$$\Theta G_{\mu\nu} \tilde{G}^{\mu\nu}$$

➤ neutron EDM + QCD sum rules/lattice QCD

$|\Theta| \lesssim 10^{-10}$

➤ fine tuning (expect $\mathcal{O}(1)$)

strong CP problem
Introduction

- CP violation beyond standard model
  - quark EDM
    \[ \sum_f \bar{\psi}_f \sigma_{\mu \nu} F^{\mu \nu} \psi_f \]
    \( F^{\mu \nu} \) - EM field strength tensor
  - quark chromo-EDM
    \[ \sum_f \bar{\psi}_f \sigma_{\mu \nu} G^{\mu \nu \gamma_5} \psi_f \]
    \( G^{\mu \nu} \) - QCD field strength tensor
  - 3-gluon (Weinberg) operator
    \[ G_{\mu \nu} G^{\mu \lambda} \tilde{G}^{\nu}_{\lambda} \]
  - 4-quark operators
Introduction

Nucleon EDM

[Slide from Hiroshi Ohki (Lattice 2019)]

Paramagnetic Atom EDM / Molecules

Diamagnetic Atom EDM

MQM

Schiff moment

N EDM

N-N int

Nuclear EDM

q EDM

q cEDM

4-q int

g g g

θ-term

BSM physics:

Higgs doublets

Supersymmetry

Left-Right

Leptoquark

Extradimension

Composite models

Standard Model

Standard Model

Energy scale

Atomic

Nuclear

Hadron

QCD

TeV

observable : Observable available at experiment

: Sizable dependence

: Weak dependence

: Matching

2.3 Sources of CP violation from BSM physics

In many scenarios of BSM, large EDMs are predicted, because of higher order contributions that can arise at the one- or two-loop levels. These contributions are overwhelmingly exceed over the loop suppressed SM contribution. In Fig. 4, we present the typical lowest order CP violating processes of BSM contributing to the EDMs at the elementary level. In this subsection, we would like to elaborate several such well motivated candidates of BSM which can generate EDMs.

Important bottleneck of the EDM calculation!

Role of (lattice) QCD: connect quark/gluon-level (effective) operators to hadron/nuclei matrix elements and interactions (Form factor, dn)

Non-perturbative determination is important \(\rightarrow\) Lattice QCD calculation!
The CoE that never was.....

Atomic/molecular

Nuclear

Nucleon

Lepton/quark

diamagnetic system (Hg, Ra, Tl, ...)

paramagnetic system (ThO, YbF, HfF+, Tl, ...)

nuclear Schiff moment

P,T-violating nucleon-nucleon interaction

quark and chromo EDMs, theta QCD angle

neutron EDM

Beyond Standard Model CP-violating phases

CI Ginges (UQ)

CI Quiney (Melbourne)

CI Simenel (ANU)

CI Zanotti (Adelaide)

CI White (Adelaide)

CI Quiney (Melbourne), CI Ginges (UQ)

CI Zanotti (Adelaide)

CI White (Adelaide)
The CoE that never was.....

Lattice QCD contribution
The Standard Model

• The three forces described by the Standard Model and the corresponding force carriers are:

  • **The Electromagnetic force** - the photon $\gamma$
    • Acts on electric charge.
    • $\gamma$ is massless so EM interaction has “infinite” range
  
  • **The Weak force** - $W^+, W^-, Z$
    • Acts on flavour.
    • $Z^0, W^{+/-}$ are very heavy, so the weak force is very short range.

  • **The Strong force** - the gluon $g$
    • Acts on “colour charge”
    • Gluon is massless, but strongly self-interacting - restricted to short distances.
    • Described by Quantum Chromodynamics (QCD)
The Standard Model

» Quantum Electrodynamics is well understood

» Precision tests - determine $\alpha_{\text{QED}}$ by fitting experimental measurements with theoretical predictions

» Low energy range, accessible with small experiments $(g - 2)_e$

» High energy range, accessible with particle colliders (e.g. $e^+e^-$ colliders)

» Condensed matter systems (quantum Hall effect, Josephson effect)

» Anomalous magnetic moment of the electron computed theoretically to $\alpha^5$

\[
\frac{g}{2} = 1 + C_2 \left( \frac{\alpha}{\pi} \right) + C_4 \left( \frac{\alpha}{\pi} \right)^2 + C_6 \left( \frac{\alpha}{\pi} \right)^3 + C_8 \left( \frac{\alpha}{\pi} \right)^4 + C_{10} \left( \frac{\alpha}{\pi} \right)^5 + \cdots + a_{\text{hadronic}} + a_{\text{weak}}
\]

$$1/\alpha = 137.035999074(44) \ [0.32 \ \text{ppb}]$$
The Standard Model

QED coupling constant is small and roughly constant over a large energy range

\[ \alpha(Q^2 = 0) \sim 1/137 \quad \alpha(Q^2 \approx M_W^2) \sim 1/128 \]

Perturbation theory is a useful tool
The Standard Model

QED coupling constant is small and roughly constant over a large energy range

\[ \alpha(Q^2 = 0) \sim 1/137 \quad \rightarrow \quad \alpha(Q^2 \approx M_W^2) \sim 1/128 \]

Perturbation theory is a useful tool

Strong coupling constant varies dramatically

Asymptotic Freedom

perturbation theory useful here
e.g. Deep-Inelastic Scattering
quarks look like “free” particles

Asymptotic Freedom
The Standard Model

But $\alpha_s \sim O(1)$ at low energies

perturbation theory no longer a useful tool

Quarks confined inside hadrons

Confinement

need a nonperturbative method

(all orders of $\alpha_s$)

Lattice QCD
Nonperturbative physics?

Why should we be interested in NP physics?

Constraining the flavour sector of the Standard Model (CKM matrix) \( \Rightarrow \) Requires BOTH experimental and NP theoretical measurements

Dynamical mass generation \( \Rightarrow \) \( M_p(uud) \approx 940 \text{ MeV} \)

But \( 2m_u + m_d \approx 10 \text{ MeV} \)

Structure of proton (e.g. form factors - see later) \( \Rightarrow \) NP quantities. Not all are accessible in experiment

Matrix elements of effective operators relevant for nEDM \( \Rightarrow \) Definitely NP

+ many more....
Lattice
The Lattice

- Work in Euclidean space $t \rightarrow i\tau$
- Discretise space-time with lattice spacing, $a$,

$$\mathbb{L} \subset a\mathbb{Z}^4 = \{x | x^\mu = an^\mu, n \in \mathbb{Z}^4\}$$

- If we have a finite lattice we usually introduce periodic boundary conditions

- ie formulate theory on the 4-torus

The Lattice

- Quark fields reside on sites $\psi(x)$
- Gauge fields on the links $U_\mu(x) = e^{-ia g A_\mu(x)}$
- Approximate the full QCD path integral by Monte Carlo methods

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}A \mathcal{D}\bar{\psi} \mathcal{D}\psi \mathcal{O}[A, \bar{\psi}, \psi] e^{-S[A, \bar{\psi}, \psi]}$$

$$\langle \mathcal{O} \rangle \approx \frac{1}{N_{\text{conf}}} \sum_i \mathcal{O}(U[i])$$

With field configurations $U_i$ distributed according to $e^{-S[U]}$

Put it on a supercomputer
Systematics of a lattice calculation

- Extrapolations:
  - Continuum
    - Unavoidable
  - Improved actions (errors $O(a^2)$)
  - Finer lattice spacings

\[
\alpha \rightarrow 0
\]
Systematics of a lattice calculation

- **Extrapolations:**
  - Continuum
  - Unavoidable
  - Improved actions (errors $O(a^2)$)
  - Finer lattice spacings
  - Finite volume
    - Large volumes so effects are exponentially suppressed

$a \rightarrow 0$

$L \rightarrow \infty$
Systematics of a lattice calculation

- **Extrapolations:**
  - Continuum
  - Unavoidable
  - Improved actions (errors $O(a^2)$)
  - Finer lattice spacings
  - Finite volume
    - Large volumes so effects are exponentially suppressed
  - Chiral
    - Chiral perturbation theory
    - Simulate at physical quark masses

\[
\alpha \rightarrow 0 \quad L \rightarrow \infty
\]

\[
m_\pi \rightarrow 140\text{MeV}
\]

\[
\text{GOR} \Rightarrow m_\pi^2 \propto m_q
\]
Quark mass dependence

Nucleon Mass

\[ m_{\pi}^2 \propto m_q \]
Speed of a Lattice Calculation

1000 configurations with L=2fm

[Ukawa (Berlin, 2001)]
Speed of a Lattice Calculation

1000 configurations with L=2fm

[Ukawa (Berlin, 2001)]

Algorithmic improvements
Speed of a Lattice Calculation

1000 configurations with L=2fm

[UKAWA (BERLIN, 2001)]

Algorithmic improvements

[CLARK (TUCSON, 2006)]
Speed of a Lattice Calculation

1000 configurations with L=2fm

[UKAWA (BERLIN, 2001)]

Algorithmic improvements

June 2001: IBM ASCI White (LLNL), 4.9 TFlops
Speed of a Lattice Calculation

1000 configurations with $L=2\text{fm}$

[Ukawa (Berlin, 2001)]

Algorithmic improvements

Faster supercomputers

June 2001: IBM ASCI White (LLNL), 4.9 TFlops
Speed of a Lattice Calculation

1000 configurations with L=2fm

[Ukawa (Berlin, 2001)]

Algorithmic improvements

Faster supercomputers

June 2001: IBM ASCI White (LLNL), 4.9 TFlops

[Clark (Tucson, 2006)]

November 2019: Summit (Oak Ridge) 148 PFlops
The Power of Computers

➤ Can we harness the power of Graphics Cards?

➤ Lattice community some of the earliest users of GPUs for scientific purposes

“Lattice QCD as a video game” [Egri, et al., hep-lat/0611022]

➤ Now widely used by many groups

➤ Specialised codes written (e.g. QUDA)

Results from TitanDev
- $48^3 \times 512$ aniso clover
- scaling up 768 GPUs

Clark, Lattice 2013
Australian computing (NCI)

➤ June 2001:
- AlphaServer SC ES40/EV67
  
  167 GFlops

➤ November 2019:
- Fujitsu Primergy CX2570 M5
- 3,200 nodes
- (Intel Cascade Lake + NVidia V100 GPUs)
  
  4.4 PFlops
- Full capacity early 2020
Real-Time Evolution of Lattice Results

Nucleon Mass

$M_N [\text{GeV}]$

$m^2_\pi [\text{GeV}^2]$
Strong CP

- Recall lattice configurations sampled according to
  \[ e^{-S_{\text{QCD}}} \quad \text{(importance sampling)} \]

- Including CP-violating \( \Theta \) term on the lattice
  \[ e^{-S_{\text{QCD}} - iQ\Theta} \]

  Destroys importance sampling \hspace{1cm} \text{(sign problem)}

- Workarounds:

  1. Expand in small values of \( \Theta \)

  \[ e^{-S_{\text{QCD}} - iQ\Theta} = e^{-S_{\text{QCD}}} [1 - iQ\Theta + \mathcal{O}(\Theta^2)] \]

  \[ \langle \mathcal{O} \rangle_{\mathbb{C}P} = \langle \mathcal{O} \rangle_{\mathbb{C}P-\text{even}} - i\Theta \langle Q\mathcal{O} \rangle_{\mathbb{C}P-\text{even}} + \mathcal{O}(\Theta^2) \]

  (also applies to other CP-violating terms cEDM, \ldots)
Extraction of $d_N$

- Two methods for extracting nEDM
  - matrix elements of EM current $\rightarrow F_3(Q^2)$ form factor

$$\langle N(p', \sigma') | J^{\mu} | N(p, \sigma) \rangle_\Theta = \bar{u}_N(p', \sigma') \left[ F_1(Q^2) \gamma^\mu + F_2(Q^2) \frac{i \sigma^{\mu \nu} q_{\nu}}{2m_N} - F_3(Q^2) \frac{\gamma_5 \sigma^{\mu \nu} q_{\nu}}{2m_N} \right] u_N(p, \sigma)$$

$$d_N = \lim_{Q^2 \to 0} \frac{F_3(Q^2)}{2m_N}$$

- energy shifts in a background electric field

$$\langle N(t) \bar{N}(0) \rangle_{\Theta, \vec{E}} \sim e^{-(m_N + d_N \vec{\sigma} \cdot \vec{E})t}$$

$d_N$ determined from spin-dependent energy shifts

apply background $\vec{E}$ field to lattice

P,T even

P,T odd
Examples

- Energy shifts from background $\vec{E}$ field [PRD78 (2008) 014503]
  - Boundary effect issues from implementation of constant $\vec{E}$ field?

$F_3(Q^2)$ from matrix elements
[arXiv:1810.0372]

Physical quark masses
  - Huge statistics required
Strong CP

➤ Workarounds:

2. Simulate at small imaginary values of theta $\Theta = i\bar{\Theta}$

$$\langle \mathcal{O} \rangle_\Theta = \int \mathcal{D}U \mathcal{O} e^{-S_{\text{QCD}} - \bar{\Theta} Q}$$

➤ [Baluni, PRD 19 (1979) 2227]: invoke axial anomaly - rotate $\Theta$ term into fermion action

$$S_{\Theta} = \bar{\Theta} \frac{m_l m_s}{2m_s + m_l} a^4 \sum_x \left( \bar{u} \gamma_5 u + \bar{d} \gamma_5 d + \bar{s} \gamma_5 s \right)$$

➤ Generate new sets of lattice gauge fields for each choice of $\bar{\Theta}$

➤ On each new ensemble, determine matrix elements

$$\langle N(p', \sigma') \mid J^\mu \mid p, \sigma \rangle_\Theta = \bar{u}_{N(p', \sigma')} \left[ F_1(Q^2) \gamma^\mu + F_2(Q^2) \frac{i \sigma^{\mu\nu} q_\nu}{2m_N} - F_3(Q^2) \frac{\gamma_5 \sigma^{\mu\nu} q_\nu}{2m_N} \right] u_N(p, \sigma)$$

[arXiv:0808.1428]
[PRL115 (2015), 062001]
Result: $\bar{\Theta} = i\Theta$

- $F_3(Q^2)$ from matrix elements

Two quark mass choices

$m_\pi \approx 465$ MeV

$m_\pi \approx 360$ MeV

Each with 2 values of $\bar{\Theta} = i\Theta$

[arXiv:0808.1428]  
[PRL115 (2015), 062001]
Quark EDMs

➤ Flavour-diagonal nucleon matrix elements of tensor operator

\[ \langle N(p, s) | \bar{q} \sigma_{\mu\nu} q | N(p, s) \rangle = g_T^q \bar{u}_N(p, s) \sigma_{\mu\nu} u_N(p, s) \]

required to quantify the quark EDM contributions to nucleon EDMs

➤ e.g. assume neutron EDM comprised entirely of qEDMs

\[ d_n = d_u^\gamma g_T^u + d_d^\gamma g_T^d + d_s^\gamma g_T^s \]

places bounds on qEDMs \( d_q^\gamma \)

e.g. PRD98 (2018) 034503

➤ Implications for BSM theories
Quark chromo-EDMs

- Allow quarks to propagate in “chromo-electric” background field
  - compute 3-point functions with operator
    \[ \sum_f \bar{\psi}_f \sigma_{\mu\nu} G^{\mu\nu} \gamma_5 \psi_f \]
    inserted everywhere
  - On top of this background field compute
    \[ \langle N(p', \sigma') | J^\mu | p, \sigma \rangle_\Theta = \bar{u}_N(p', \sigma') \left[ F_1(Q^2)\gamma^\mu + F_2(Q^2) \frac{i\sigma^{\mu\nu} q_\nu}{2m_N} - F_3(Q^2) \frac{\gamma_5 \sigma^{\mu\nu} q_\nu}{2m_N} \right] u_N(p, \sigma) \]

[arXiv:1810.10301]
[arXiv:1812.06233]
Weinberg operator

- gluon chromoEDM ("Weinberg operator") $G_{\mu\nu}G^{\mu\lambda}\tilde{G}^\nu_{\lambda}$
- A challenge on the lattice
  - statistical signal for 2 gluon (topological charge) difficult
  - now 3 gluons
- renormalisation

Figure 6. Corrections suggested by [21] for the CP-odd vector form factor $F_3/2m_N$ in units of $\sqrt{\pi}$ for the neutron (left) and proton (right), induced via the $\sqrt{\pi}$-term CP-violating vacuum, plotted against transfer momentum $Q^2$. Blue and red points are the $m_\pi=411, 701\,\text{MeV}$ results respectively. The linear fit to $Q^2=0$ is used to extract the value for the neutron and proton EDM $d_n/p$.

Figure 7. Corrections suggested by [21] CP-odd vector form factor $F_3/2m_N$ in units of $\sqrt{\pi}/\sqrt{\lambda}$ for the neutron (left) and proton (right), induced via the Weinberg operator CP-violating vacuum, plotted against transfer momentum $Q^2$. Blue and red points are the $m_\pi=411, 701\,\text{MeV}$ results respectively. The linear fit to $Q^2=0$ is used to extract the value for the neutron and proton EDM $d_n/p$.

5.3 Conclusion
In this proceeding, we have obtained preliminary results for the $\sqrt{\pi}$ and Weinberg EDM with $N_f=2+1$ dynamical gauge configurations at 2 pion masses. We have defined the CP-odd local operators using the gradient flow for the gauge fields. We have treated the CP-sources in a perturbative manner allowing us to use existing QCD gauge configurations. For the $\sqrt{\pi}$-EDM we clearly see a signal at the heavier pion mass while the lightest pion mass is still consistent with zero. Both the sign of the EDMs and the slopes in $Q^2$ of the CP-odd form factors are consistent with $\mathcal{PT}$. For the Weinberg EDM we see a clear signal at both pion masses, but the observed pion-mass dependence is not was we expected [arXiv:1711.04730].
Summary

➤ Possible to study SM and BSM contributions to nEDM on lattice
  ➤ $\Theta$ term
  ➤ quark EDM
  ➤ quark chromo-EDM
  ➤ 3-gluon (Weinberg) operator

➤ Early days, but now being addressed by several international collaborations
  ➤ Australian effort?
    - imaginary $\Theta$ - current
    - qEDM, cEDM, N-N - future?

➤ Much work still needed, even for $\Theta$ term
  ➤ statistical errors (brute for force or better ideas?)
  ➤ lattice systematics