HEATING TOKAMAK PLASMA WITH FAST IONS

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OUTLINE OF LECTURE 2A

• Ohmic heating of plasmas

• Evolution of fast ion distribution function in energy space, ion and electron heating

• Drift orbits

• Neutral beam injection (NBI)

• Ion Cyclotron Resonance Heating (ICRH)

• Fusion products

• Summary
OHMIC HEATING

- All tokamaks are heated initially by the plasma current
  Ohmic power = $I_p \times V = [I_p]^2 \times R$
  Plasma resistivity $R \sim [T_e]^{-3/2}$

- As the plasma gets hotter:
  - its resistivity gets smaller – the ohmic power falls
  - the energy losses increase - $\tau_E$ gets smaller
  - there is a maximum temperature $\sim 5 \text{ keV}$ that can be reached by Ohmic heating

- Additional heating techniques are needed to obtain 10-20 keV temperature thermal ions. Heating plasma up to this temperature range with a small density population of “fast” ions with energy $E_f >> T_i \sim T_e$ is the most attractive and natural way
HEATING THE PLASMA WITH FAST ION POPULATION

- Fast particle population (e.g. fusion-born alpha-particles) has high energy,

\[ E_f \gg T_i \sim T_e \]

but low density,

\[ n_f \ll n_e \]

- Energy content of the fast particle population may be comparable to thermal plasma energy content,

\[ \beta_f = n_f E_f \sim \beta_{therm} \]

- Source of the fast ions may be isotropic in velocity space (fusion-born alpha-particles) or anisotropic (a beam of fast ions)

- The fast ions transfer their energy to thermal ions and electrons by Coulomb collisions.
Consider evolution of fast ion distribution function, \( f(v_x, v_y, v_z) \), due to the Coulomb collisions with thermal plasma species in homogeneous plasma. An axial symmetry of the distribution function is assumed during the evolution:

\[
f(v_x, v_y, v_z) = f(v, \vartheta) ; \quad v = \sqrt{v_x^2 + v_z^2} ; \quad \vartheta = \tan^{-1}(v_z/v_x) ; \quad v_\perp = \sqrt{v_y^2 + v_z^2} ; \quad v_\parallel = v_x
\]

This evolution can be described by Fokker-Planck equation for fast ion distribution function \( f(v, \vartheta) \) with initial velocity \( V_0 \).

Typical range of the fast ion velocities satisfies \( v_i \ll V_0 \ll v_e \). Neglecting self-collisions between fast ions, Fokker-Planck equation becomes linear and it has the form.
FAST ION EVOLUTION IN MAXWELLIAN PLASMA-2

\[ \frac{\partial f}{\partial t} = \frac{V_0^3}{\tau v^2} \left\{ \frac{\partial}{\partial v} \left[ \frac{V_0^2 a(v)}{2v} \frac{\partial f}{\partial v} + b(v) f \right] + \frac{c(v)}{V_0} \frac{1}{\sin \vartheta} \frac{\partial}{\partial \vartheta} \left[ \sin \vartheta \frac{\partial f}{\partial \vartheta} \right] \right\} - v f + pF \]

where

\[ a(v) = \frac{T_e}{E_0} \left\{ \tilde{Z}_2 + \frac{4}{3\sqrt{\pi}} \frac{m_0}{m_e} \left( \frac{v}{v_e} \right)^3 \right\} \; ; \; \; b(v) = \tilde{Z}_1 + \frac{4}{3\sqrt{\pi}} \frac{m_0}{m_e} \left( \frac{v}{v_e} \right)^3 \; ; \; \; c(v) = Z_{\text{eff}} \frac{V_0}{2v} + \frac{2}{3\sqrt{\pi}} \frac{V_0}{v_e} = Z_{\text{eff}} \frac{V_0}{2v} \left( 1 + \frac{4}{3\sqrt{\pi}} \frac{v}{v_e} \right) \; ; \]

\[ \tau = \frac{1}{\pi \sqrt{2} Z_b e^4 n_e L} \]

\[ \tilde{Z}_1 = \frac{m_0}{n_e} \sum_i \frac{Z_i^2 n_i}{m_i} \; ; \; \; \; \tilde{Z}_2 = \frac{m_0}{n_e T_e} \sum_i \frac{Z_i^2 n_i T_i}{m_i} \; ; \; \; \; Z_{\text{eff}} = \left( \frac{1}{n_e} \right) \sum_i Z_i^2 n_i \; ; \]

\( L \) is the Coulomb logarithm, \( m_0 \) the fast ion mass, \( E_0, V_0 \) are the initial energy and velocity of fast ions and two last terms in the right-hand-side of this equation represent sink and source of the energetic ions. Source: \[ 2\pi \int_0^\infty v^2 dv \int_0^\pi F(v - v_0; \vartheta) d\vartheta = 1 \]

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VELOCITY SPACE DIFFUSION AND DRAG

\[
\frac{\partial f}{\partial t} = \frac{V_0^3}{\tau v^2} \left\{ \frac{\partial}{\partial v} \left[ \frac{V_0^2 a(v)}{2v} \frac{\partial f}{\partial v} + b(v) f \right] + \frac{c(v)}{V_0} \cdot \frac{1}{\sin \vartheta} \frac{\partial}{\partial \vartheta} \left[ \sin \vartheta \frac{\partial f}{\partial \vartheta} \right] \right\} - v f + pF
\]

- Diffusion of \( f(v, \vartheta) \) is given by two terms proportional to \( a(v) \) and \( c(v) \), with the pitch-angle scattering \( c(v) \) usually dominating over the velocity diffusion \( a(v) \).
- The diffusion gives a broadening (in velocity space) of the distribution function.
- The drag (also called "dynamical friction") is represented by \( b(v) \).
- The drag gives a deceleration (slowing-down) of the distribution function.
CLASSICAL SCHEME OF PLASMA HEATING BY FAST IONS-1

- The fast ions transfer their energy to thermal ions and electrons by Coulomb collisions. If the energy of the fast ions is less than a critical value

\[ E_{\text{crit}} = 14.8 A_f T_e \left( \sum_i n_i Z_i^2 / n_e A_i \right)^{2/3} \]

power flows mainly to thermal ions rather than to electrons. Here, \( A_f, A_i \) are atomic masses of fast ions and thermal ions, \( T_e \) is electron temperature, \( n_i \) and \( n_e \) are ion and electron densities, and \( Z_i \) is atomic number of thermal ions.
- Above the critical energy, the drag dominates in Fokker-Planck equation
- Below the critical energy the diffusion dominates
The amount of energy going from ions with initial energy $E$ into plasma ions is given by Stix formula $G_i = \frac{E_{\text{crit}}}{E} \int_0^{E/E_{\text{crit}}} \frac{dy}{1 + y^{3/2}}$, and $G_i(E/E_{\text{crit}})$ is illustrated below.
ION ORBITS IN A TORUS - 1

- In a straight magnetic field the ions move along helical orbits centered on a field line. The radius of the orbit is the Larmor radius.

- In a torus the ion orbits are centered on drift surfaces which are displaced from the magnetic surface by value $\Delta_{\text{Orbit}} \equiv \Delta_O$.

- The Larmor radius depends on $B_T$. The displacement $\Delta_O$ depends on $B_P$ (and thus on current $I_P$) and is:
  - Inwards for ions moving in the same direction as $I_P$.
  - Outwards for ions moving in the opposite direction.
ION ORBITS IN A TORUS - 2

- Some ions are reflected by the strong toroidal field at the inside of the torus
- The projection of the ion orbit onto the poloidal plane looks like a “banana”

[Diagram showing a particle orbiting in a toroidal field, with annotations for R, d, and the banana orbit.]
FAST PARTICLE ORBITS: TRAPPED ORBITS

Projection of poloidally trapped ion trajectory

Toroidal direction $\omega_\phi$

Fast ion trajectory

Poloidal direction $\omega_\theta$

Ion gyro-motion $\omega_{ci}$

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NEUTRAL BEAM INJECTION - 1

- Ions from the ion source accelerate by grids to high energy
- Then they pass through the neutraliser and become neutral high energy atoms
- The neutral beam penetrates the tokamak magnetic fields. The penetration of the beam depends on the NBI energy, mass and on the plasma density
- Within plasma neutrals are ionized by collisions with thermal ions & electrons
- These fast ions are trapped by the tokamak magnetic fields
- NBI systems on JET, JT-60U, TFTR, DIII-D have $E \leq E_{\text{crit}}$ so they heat IONS
- NBI systems on MAST & NSTX, (and future NBI on ITER) have $E > E_{\text{crit}}$ so they heat ELECTRONS
Advantages

- Efficient heating of ions
- High power capability (40 MW on TFTR, 24 MW on JET)
- Drives plasma rotation (stabilising lock modes)
- Fuelling!
- Some current drive

Disadvantages

- Need MeV energy beams for penetrating in a reactor → Negative ion source for NBI is needed
- Heating not well localised
- Large aperture
ION CYCLOTRON RESONANCE HEATING - 1

\[ B \propto \frac{1}{R} \]

Absorption region

\[ \omega = \omega_{BH} \]

ICRH antenna
Advantages

- Localised heating
- Hydrogen minority ICRH creates H minority with \( E > E_{\text{crit}} \) - it heats ELECTRONS
- However, heating of IONS is also possible (e.g. \(^3\text{He}\) minority in DT plasma)
- Some current drive

Disadvantages

- Antenna inside the vessel
- Low power capability
- Plasma coupling may be a problem in, e.g. H-mode with ELMs
ALPHA PARTICLE HEATING AND BURNING PLASMAS

- Burning plasmas: auxiliary heating used, but significant plasma self-heating by fusion alphas exists → plasma becomes exothermic medium

- Alphas born at 3.5 MeV have $E > E_{\text{crit}}$ → they mostly heat ELECTRONS

- However, electrons do not fuse, we need to heat ions with fusion born alpha-particles. The problem of re-directing the alpha-heating from electrons to IONS is called “alpha-channeling”
### FAST PARTICLES IN JET DT DISCHARGE WITH 16 MW FUSION

<table>
<thead>
<tr>
<th></th>
<th>$E$, keV</th>
<th>$E_{\text{crit}}/T_e$</th>
<th>$E/E_{\text{crit}}$ for $T_e=14$ keV</th>
<th>$G_i/G_e$ = $G_i/(1-G_i)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fusion alpha-particles</td>
<td>$3.52 \cdot 10^3$</td>
<td>33</td>
<td>7.62</td>
<td>0.3</td>
</tr>
<tr>
<td>Deuterium NBI</td>
<td>140</td>
<td>16.5</td>
<td>0.61</td>
<td>5.67</td>
</tr>
<tr>
<td>Tritium NBI</td>
<td>160</td>
<td>25</td>
<td>0.46</td>
<td>9</td>
</tr>
<tr>
<td>ICRH-accelerated hydrogen</td>
<td>$\approx 500$</td>
<td>8.25</td>
<td>4.33</td>
<td>0.54</td>
</tr>
</tbody>
</table>

Main types of energetic ions in JET D-T plasma (D:T=50:50, JET pulse #42976).
### FAST PARTICLES IN JET

<table>
<thead>
<tr>
<th>Machine</th>
<th>JET</th>
<th>JET</th>
<th>JET</th>
<th>JET</th>
<th>ITER</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type of fast ions</td>
<td>Hydrogen</td>
<td>He³</td>
<td>He⁴</td>
<td>Alpha</td>
<td>Alpha</td>
</tr>
<tr>
<td>Source</td>
<td>ICRH tail</td>
<td>ICRH tail</td>
<td>ICRH tail</td>
<td>Fusion</td>
<td>Fusion</td>
</tr>
<tr>
<td>Mechanism</td>
<td>minority</td>
<td>minority</td>
<td>3rd harmonic of NBI</td>
<td>DT nuclear</td>
<td>DT nuclear</td>
</tr>
<tr>
<td>( \tau_S (s) )</td>
<td>1.0</td>
<td>0.9</td>
<td>0.4</td>
<td>1.0</td>
<td>0.8</td>
</tr>
<tr>
<td>( P_f (0) ) (MW/m³)</td>
<td>0.8</td>
<td>1.0</td>
<td>0.5</td>
<td>0.12</td>
<td>0.55</td>
</tr>
<tr>
<td>( n_f (0) / n_e (0) ) (%)</td>
<td>1.0</td>
<td>1.5</td>
<td>1.5</td>
<td>0.44</td>
<td>0.85</td>
</tr>
<tr>
<td>( \beta_f (0) ) (%)</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>0.7</td>
<td>1.2</td>
</tr>
<tr>
<td>( \langle \beta_f \rangle ) (%)</td>
<td>0.25</td>
<td>0.3</td>
<td>0.3</td>
<td>0.12</td>
<td>0.3</td>
</tr>
<tr>
<td>( \max</td>
<td>R \nabla \beta_f</td>
<td>) (%)</td>
<td>( \approx 5 )</td>
<td>( \approx 5 )</td>
<td>5</td>
</tr>
</tbody>
</table>

Slowing down time: \( \tau_S \); heating power per volume at the magnetic axis: \( P_f (0) \); ratio of the on-axis fast ion density to electron density: \( n_f (0) / n_e (0) \); on-axis fast ion beta: \( \beta_f (0) \); volume-averaged fast ion beta: \( \langle \beta_f \rangle \); normalised radial gradient of fast ion beta, \( \max | R \nabla \beta_f | \). Predicted values of similar parameters are also given for alpha-particles in ITER.

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SUMMARY

- Ohmic heating has maximum 5 keV temperature; heating by small population of fast ions is required to obtain 10-20 keV optimum for DT.

- There is a critical energy $E_{\text{crit}}$ at which the heating flows from fast ions to electrons and ions become equal. Fast ions with $E > E_{\text{crit}}$ heat mostly electrons, $E < E_{\text{crit}}$ - heat ions.

- Two dominant effects determine evolution of fast ions in velocity space: drag and diffusion.

- Drift orbits of fast ions may be passing or trapped.

- NBI and ICRH are two main techniques generating various types of fast ions.

- Alpha particles mostly heat electrons.