Using Bayesian Analysis to Study Tokamak Plasma Equilibrium

Outline

- Motivation
- Brief overview of Bayes’ Formula
- Using Bayesian analysis to study fusion plasmas via the MINERVA framework
- Current tomography results obtained for discharges on the Mega-Ampere Spherical Tokamak (MAST)
- Future plans for using Bayesian Analysis to study fusion plasmas
Why use Bayesian Analysis to Study Fusion Plasmas?

- Inference of localised density, temperature and current profiles in a fusion plasma from diagnostic data constitutes a grossly under-determined problem that is difficult to handle with current techniques without using strong underlying assumptions.
- Can give reliable statistical results for intrinsically underdetermined problems.
- Excels at allowing for the straight-forward comparison of different physical models.
- Influence of prior assumptions on final inference lessens with the addition of more data.
Background: Bayes’ Formula

- Is a probabilistic technique. Using $P$ to denote a probability distribution:

  $H = \text{Hypothesis (Model Parameters)}$
  
  $D = \text{Data (Observations)}$

  $P(H|D) = \frac{P(D|H) \cdot P(H)}{P(D)}$

- $P(H|D)$ is the **Posterior**, $P(D|H)$ is the **Likelihood**, $P(H)$ is the **Prior**, $P(D)$ is called the **Evidence**

- By using a resulting posterior as a new prior, an iterative process is formed that can incorporate all data and prior knowledge into a final posterior.
The MINERVA Framework

- Written by Jakob Svensson (IPP Greifswald)
- A framework written in Java to perform Bayesian Inference on data coming from complex systems like the ones encountered in fusion research.
Model the plasma current as a cluster of rectangular, axis-symmetric current beams; it is the current **distribution** for each of these plasma beams that we wish to infer.

**Prior Assumptions:**

- Current distributions for beams are Gaussian
- The peak of each distribution is correlated to that of its neighbours. The strength of this correlation is controlled via a hyper-parameter $\tau$
Bayesian Equivalent of Feynman Diagrams
Putting it all together

- Magnetic Predictions:
  \[ \text{Pred}_j(i) = B \text{ (or } A) = M_{ij} \cdot I_i + C_j \]

- MSE Predictions:
  \[ \text{Pred}_j(i) = \tan \gamma = \frac{A_0 \cdot B_Z(i) + A_1 \cdot B_R(i)}{A_5 \cdot B_\phi(i)} \]

- Bayes’ Formula:
  \[ \text{Posterior}(i) \propto \left( \prod_j N \left( \text{Obs}_j - \text{Pred}_j(i); \sigma_j \right) \right) \cdot \text{Prior}(i) \]
Current Tomography: Expectation of Beam Currents Give Flux Surfaces

- LCFS comparison with EFIT data over current tomography and Flux Surface Plots
- Results inferred from Magnetics & MSE Data
- Poloidal flux surfaces plotted for currents corresponding to the expectation of the posterior

Pulse = 22254
Time = 350ms
\( \tau = 200A \)
Current Tomography: Variance of Current Distributions
Show Flux Surface Uncertainties

- Plots are of Flux Surfaces corresponding to 200 beam current samples from the posterior

Pulse = 22254
Time = 350ms
\( \tau = 200A \)
Loosely, $q$ is a measure of the pitch of magnetic field lines.

Important for plasma stability.

Plot is of $q$-profiles corresponding to 200 beam current samples from the posterior.

Pulse = 22254
Time = 350ms
$\tau = 200A$
Where we are going

- Developing Bayesian analytic techniques to validate, compare, and modify various force balance models.
  - High-energy particle population corrections to Grad-Shrafranov
  - Bayesian version of EFIT: EFIT + flux surface uncertainties
- Developing Bayesian inference framework for mode structure on H-1. Ultimately, this could be used to guide the development of new physics of 3D MHD wave analysis.
- Development of Bayesian inference techniques for diagnostic optimisation.
- Development of real-time control codes via linearised, analytic models or more-advanced inversion methods.
  - Priors trained on historical data
  - Advanced sampling algorithms
Wider Possibilities for Experiments

- Combining Different diagnostics to infer hard-to-measure quantities (e.g. electric field).
- Deployment of MINERVA to other experiments beyond JET, MAST and H-1 (e.g. ITER).
Conclusions

- MAST Magnetics and MSE have been successfully used to infer current tomography for MAST discharges using Bayesian analysis techniques via the MINERVA framework.
- Thompson Scattering has been integrated into the current tomography analysis to infer electron temperature and density profiles.
- Spin-off: Bayesian Analysis methods (e.g. Tikhonov cross-validation) have been developed to quantitatively identify problematic magnetic diagnostics on MAST and effectively handle outlier data.
References

