Detection, analysis, interpretation, and theory of high frequency MHD activity in MAST and K-STAR

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Outline of Capabilities

• Detailed MHD equilibrium ($\delta W$) reconstruction: essential for MHD wave analysis.
  — Kinetic reconstruction
  — Bayesian analysis (see talk by Greg. von Nessi)
• ideal MHD stability ($\delta^2 W$)
  — Marginal stability to low $n$ and ballooning modes ($n=\infty$)
  — Structure of sawteeth modes
• Resolving energetic components on $\delta W$, $\delta^2 W$: Beyond MHD
• Precision Mirnov array design & installment
• Sophisticated mode analysis techniques: Fourier-SVD (optimal extraction, optimal positioning), Bicoherence analysis.
• Wave analysis and interpretation: GAEs, Alfven gap modes, CAEs, mode analysis, wave-mode drive, phase coupling
• K-STAR collaboration on electron fishbones / sawteeth
• Collaboration summary and ideas
MAST, A~1.4

**MAST’s mission:**
- to explore the ST concept
- test conventional aspect ratio physics

**Aspect ratio (A=R/a):**
- A=4.5
- 3.0
- 1.25

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Baseline</th>
<th>Achieved (2009)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Major Radius</td>
<td>0.85 m</td>
<td>0.85 m</td>
</tr>
<tr>
<td>Minor Radius</td>
<td>0.65 m</td>
<td>0.65 m</td>
</tr>
<tr>
<td>Elongation</td>
<td>2.5</td>
<td>2.5</td>
</tr>
<tr>
<td>Triangularity</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Plasma Current</td>
<td>2 MA</td>
<td>1.5 MA</td>
</tr>
<tr>
<td>Toroidal Field</td>
<td>0.51 T</td>
<td>0.62 T</td>
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<tr>
<td>NBI Heating</td>
<td>5 MW</td>
<td>3.8 MW</td>
</tr>
<tr>
<td>RF Heating</td>
<td>1.5 MW</td>
<td>1 MW</td>
</tr>
<tr>
<td>Pulse Length</td>
<td>5 sec</td>
<td>&gt;0.5 sec</td>
</tr>
</tbody>
</table>
MHD Equilibrium Reconstruction

- Magnetic reconstruction (e.g. EFIT)
  - External sensors measure $B$, $I_p$
  - assume forms of $p(\psi)$ and $f(\psi)$

- Kinetic refinement (iteration):
  constrain $p(\psi)$ to data
  
  (1) $T_e$, $n_e$ profile (300 chord) from Thomson Scattering

  (2) $T_i$, $n_i$ profile
  $Z_{\text{eff}}$ profile (200 chord) from visible Bremsstrahlung radiation:
  impurity analysis $\rightarrow n_i$
  $T_i$, $v_i$ from Charge Exchange recombination spectroscopy

  (3) $P_{fast}$ gyro-kinetic simulations $\rightarrow$ fast-ion distribution
  resulting from collisions with injected neutrals.

Folding in TS data (electrons)

- Take separatrix and $J_{\parallel}(\psi)$ from EFIT

#7085 at 290ms

D plasma, 2.75MW D-NBI

$I_p \sim 260kA$

$T_e \sim 700eV$

No disruption

(1) Obtain $n_e$, $T_e$ profile from measurements, and map to $\psi$
Folding in Bremsstrahlung, CXR data (ions)

(2) Ion data:
visible Bremsstrahlung gives $Z_{\text{eff}}$ → (assume fully stripped C)
$n_i = (0.78 \pm 0.05) \, n_e$
Charge exchange gives $T_i = (1.5 \pm 0.15) \, T_e$

- Estimate thermal pressure
  \[
p \approx n_i k_B T_i + n_e k_B T_e
  \]

- Use fixed boundary, high resolution GS solver CHEASE[1] to reconverge equilibrium, whilst remapping
  \[
  \tilde{I}_\| (\psi_n) \rightarrow I_\| (\psi_n^t)
  \]
  to conserve plasma current $I_p$

\[<Z_{\text{eff}}>= 2 \pm 0.2\]

\[T_i, T_e \, (\text{keV})\]

7085 at 290ms

\[\frac{p}{\text{[kPa]}}\]

Add $P_{fast}$ estimated from NBI simulation

$q_0=2.4, \quad \beta_N=6.37,$

$q_{95}=13.9, \quad \beta_t=6.24,$

$q_{100}=33.1, \quad I_N=0.98,$

$l_I=0.94 \quad p(0)/<p>=2.65,$

$E_{th}=\frac{3}{2}\int p dV=52.7 \text{ kJ}$
MHD scans

- equilibrium + stability scans driven by Matlab and Python scripts
MHD scans

- equilibrium + stability scans driven by Matlab and Python scripts
- $n<10$, KINX [1]
- $n=\infty$, BALM [2]

**MAST n=1 stability**

- MAST #7085 near no-wall marginal stability on $\beta_n$ vs $l_i$ curve

- Finite resolution model of ideal wall

- With wall MAST limit ~10% above no wall limit, as MAST has poloidal field coils in tank (like H1)
Beyond MHD: energetic-resolved fluids

- Idea: Include energetic particles (e.g. Neutral beam, rf) as a separate fluid

- Single fluid flow MHD equations generalised to multiple energetically resolved Gaussian fluids: each fluid with unique $T$, $n$, $v$,

- Extend single-fluid code FLOW to multiple species (FLOW-M): solve equilibrium with multiple energetic components.

- New ARC project to extend to mode structure and stability (with CCFE)

Hole and Dennis, Plasma Phys. Control. Fusion 51 (2009) 035014
Energetic particle driven modes

• In STs
  – the low $B$ field makes it easy to obtain a super-Alfvénic population
  – large aspect ratio $\Rightarrow$ large gap in the continuum of singular frequencies

• Various high frequency MHD activity
  – TAEs, NAEs [e.g. LC Appel, M Hole, R J Akers, S D Pinches Varenna Theory Conf. 2002].
  – EPMs

Pre 2004: no dedicated high frequency ($100\text{kHz} < f$) $\delta B$ coils
Challenge: model, design and build a new high-$f$ array
New Mirnov Coils: Modelling and Design

- **Diagnostic Transfer Function:**
  \[
  \frac{V_f}{V_p} = H_f(\omega)H_a(\omega)H_p(\omega)
  \]

- \( V_p = NA \frac{dB_c}{dt} \)

- Graphite shield

- Graphite coated centre-column

- Amplifier

- A/D converter

- MV coil/ transmission line

- Plasma

- Plasma

- Plasma

- Plasma

- Plasma

- Plasma

- Plasma

- Plasma

- Plasma

- Plasma
Design objectives for high $f$, $\delta B$ array

Outboard Mirnov Array for High-frequency data Acquisition

1. first self-resonant frequency of Mirnov coil, $f_0 > 5$ MHz,
2. detected voltage maximised at 100kHz,
3. minimized variance of the transfer function,
4. impedance matched,
5. toroidal mode number identification up to $n_c = 20$, and
6. error in toroidal mode number at $n_c = 20$ less than 5.

- Extension of remote calibration technique to account for transmission line effects
  Appel and Hole, Rev. Sci. Instrum. 76, 093505 2005
- Calculation of self-resonant frequency limits of solenoids
- New technique for mode analysis – optimum placement
Hole, Appel, Martin,
*REVIEW OF SCIENTIFIC INSTRUMENTS* 80, 123507 2009
Example: CAE activity in #9429

- #9429: D beam heated D plasma.

CAE’s believed to aliased down from 1.4 MHz to 1.9 MHz.

High resolution TS time:

\[ n_e = 6.8 \pm 0.1 \times 10^{-19} \text{ m}^{-3}, \]
\[ T_e = 0.8 \pm 0.04 \text{ keV}, \]
\[ B_\phi = 0.46 \text{ T (EFIT)} \]

Sawteeth (\( q_0 < 1 \))
Fourier-SVD analysis \( \Rightarrow \) modes numbers

Probability that signal phase pattern was generated by noise

Fourier-SVD:
- Multiple modes
- Entire array used synchronously
- Statistics of fit intrinsic

Fine-scale $n$ & $f$ splitting

166ms

210ms

Spectrum at 210ms

Frequency correlation

- Same $\Delta n=1$ sideband splitting
- Same frequency splitting

Nonlinear modulation model

- Add CAE frequency to tearing mode, and drive nonlinearly

\[ x_p = A_1 \cos(\omega_1 t) + A_2 \cos(\omega_2 t + \Phi_2) + x_{\text{noise}} \]

- Drive nonlinearly

\[ y_{NL}(\beta, x_p, x_{\text{noise}}) = \beta_1 x_p + \beta_2 x_p^2 + \beta_3 x_p^3 + \beta_4 x_p^4 + \beta_5 x_p^5 + x_{\text{noise}} \]

- Simulated frequency response:
  - frequencies agree
  - amplitudes agree

What about phase?

![Graph showing frequency response](image)
Bispectrum suggests phase coupling

Bicoherence $b(f_k, f_l)$

$$B(f_k, f_l) = \left\langle X_i(f_k)X_i(f_l)X_i^*(f_k + f_l) \right\rangle$$

$$b^2(f_k, f_l) = \frac{|B(f_1, f_2)|^2}{\left\langle |X(f_k)X(f_l)|^2 \right\rangle \left\langle |X(f_k + f_l)|^2 \right\rangle}$$

If amplifier assumed to have cubic nonlinearity...

Biphase along chords of bispectrum

...phases agree.
POSTECH, K-STAR, ANU collaboration

  - Determine whether electron fishbones can be excited in K-STAR by ECRH injection
  - Identify experimental parameters for maximum mode excitation

- Shots #2170-#2180. Highest current shot #2179 with \( I_p = 260\text{kA}, \) \( P_{ECRH} = 260\text{kW}, \) \( n_e \sim 4 \times 10^{19} \text{ m}^{-3}. \)

- Preliminary findings: lots of sawteeth (~100Hz)
- Faint 4-6kHz mode, possible (\( m,n \)) = (8,3)
- Ongoing analysis as to what it is…
Capability Summary

• Detailed “kinetic” and “Bayesian” equilibrium reconstruction
• Ideal MHD stability (low n + ballooning, linear mode structure of sawtooth)
• Energetic particle resolution: Beyond MHD.
• Wave analysis and interpretation: GAEs, Alfven gap modes, CAEs, mode analysis, wave-mode drive, phase coupling
• Precision Mirnov array: design/development of high frequency magnetic fluctuation - unprecedented resolving power.
• Sophisticated mode analysis techniques: Fourier-SVD (optimal extraction, optimal positioning), Bicoherence analysis.
• Electron fishbones: collaboration with POSTECH on K-STAR
...and some collaboration ideas

• High frequency MHD array – can equilibrium pickup coils be pulled out of integrating mode?
• Detailed equilibrium modelling techniques, including Bayesian inference: project led by M. Hole, funded by ISL CG130047 (see talk by von Nessi).
• Plasma control: interest from Boeing.
• TAE antenna?